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OPTIMIZING THE COMPRESSIVE STRENGTH OF CEMENT MORTAR (1:3) USING SCHEFFE'S SIMPLEX LATTICE THEORY

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Abstract

In this research study, a mathematical model is developed to optimize the compressive strength of cement mortar using Scheffe's simplex-lattice theory. The research work investigates the use of experimental design and statistical approaches in the optimization of mortar compressive strength. The experimental design was carried out in the laboratory using a total of 33 cubes. The optimized compressive strength of mortar (mix ratio, 1:3) was found to be 17.50N/mm^2 at a corresponding optimized ratio, 1:2.60. Two statistical models (i.e quadratic and special cubic) were developed using the scheffe's lattice theory, based on the data obtained from the experimental data, facilitating the study on the influence of mortar mix proportioning on its compressive strength by comparing the experimental value with the predicted value. It was recommended that a more generalized model should be formulated, which can give an optimized compressive strength for a wider range of mix ratios, hence other softwares like MINITAB, MATLAB, SAS, etc. may be employed in the analysis of this work and the results compared.

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Introduction

Background of the study

Mortar is a workable paste used to bind building blocks such as stones, bricks and concrete masonry units together, fill and seal the irregular gaps between them, and sometimes add decorative colors or patterns in masonry walls. In the broadest sense mortar includes pitch, asphalt, soft mud or clay, etc.

Types of mortar includes; ancient mortar, ordinary Portland cement mortar, polymer cement mortar, lime mortar, pozzolanic cement, firestop cement, etc.

Blended cement has become quite popular in developed countries, due to its durability and high benefits / cost ratio, (Anyaogu et al, 2013).

Cement mortar is a paste prepared by adding required quantity of water to a mixture of binding material like cement, and sand. This becomes hard when it is cured, resulting in a rigid aggregate structure. The binding material and sand or fine aggregates are sometimes referred to as matrix and adulterant. The durability, strength and quality of the matrix and the combined effect of the two components of mortar is able to bind the bricks or stone firmly.

Optimization means finding an alternative with the most cost effective or highest achievable performance under the given constraint, by maximizing undesired ones. Maximization means trying to attain the highest or maximum result or outcome, sometimes, without regard to cost or expense.

The mixture approach, takes the total amount (mass or volume) of the product to be fixed, and the settings of each of the *q* components are proportions, since the total amount is constrained to sum to one, (Okere et. al, 2013), only *q*-1 of the factors (component variables) can be chosen independently.

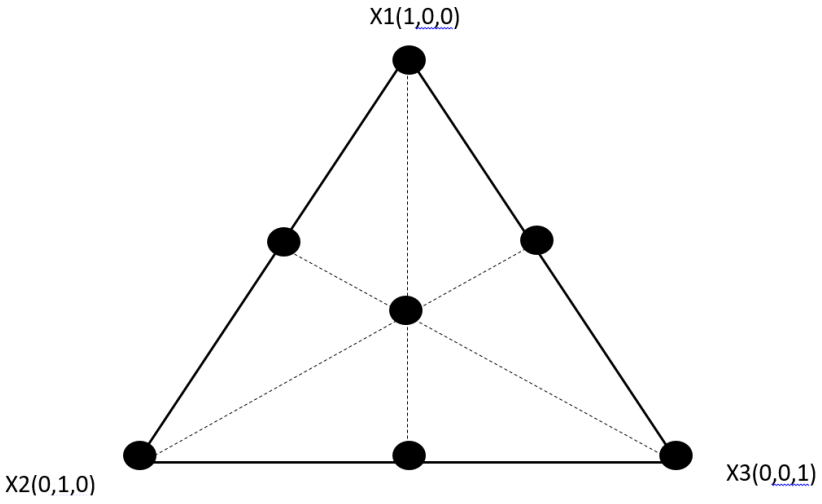


Fig 1: Simplex-Centroid Design for Three Variables

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Consider mortar as a simple (hypothetical) example of a mixture experiment, comprising three components; Water (X_1), Cement (X_2) and Sand (X_3), where xi represents the volume/mass fraction of a component. The volume fraction of the components sum to one, and the region defined by this constraint is the regular triangle (simplex) shown in Fig. 1

Problem statement

Obtaining the proper mix proportion to achieve strength is becoming more difficult and useful tools such as statistical and experimental methods are needed to aid the optimization of mortar strength.

Aim

The aim of this research work is to investigate the use of experimental design and statistical approaches in optimization of mortar compressive strength.

Objectives

The objectives of this research are:

1. To develop a statistical model from experimental data.
2. To optimize the mix proportioning for available material.
3. To study the influence of mortar mix proportioning on its proportioning.

LITERATURE REVIEW

Imoh C. A. et al (2022) researched Soil materials with poor engineering performance such as those rich in clay are prevalent across the globe. The clayey nature of these soils is of one the factors responsible for the low and excess volumetric changes in the soil which constitutes the foundation failures of most civil engineering infrastructures. For that reason, this present-day study demonstrates practically on optimizing additive for the purpose of ameliorating black expansive clay soil (ECS) and lateritic soil (LS) materials, respectively. Thus, a total of twelve (12) different mix combinations of water, soil and cement kiln dust (CKD) for both (ECS and LS) based on Scheffe's strategy were taken into consideration for the experimental protocols. Interestingly, the experimental response included California bearing ratio and unconfined compressive strength. Based on the Scheffe design of experiment, the most favourable outcome was recorded at 1.0:0.9:0.3 for soil, CKD and water, respectively. Based on the upshots, statistical indicators such as t-test and ANOVA were deployed in testing the formulated models' adequacy. In addition, qualitative test such as scanning electron micrographs and fibermetric analysis of the virgin soil materials were at variance with soil materials altered with additive.

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Furthermore, these variations are indicator towards the build-ups of cementitious compounds and this confirms clearly the feasibility of reusing CKD combined with Scheffe optimisation approach in soil re-engineering studies.

Edidiong E. A. et al (2021) showed in a research that the Ceramic industry generates a large amount of wastes which are presently not reused in any significant quantity. Reusing these wastes in concrete could solve the ceramic industry waste management problem and also lead to a more sustainable concrete industry. While the use of ceramic wastes as coarse aggregate has been extensively investigated, not much findings are available on its use as fine aggregate and there are presently no models for predicting the properties of ceramic wastes aggregate concretes. This study investigates effect of crushed recycled-ceramic tiles (CRT) fine aggregate content on compressive strength of concrete. Scheffe's second degree polynomial models were also formulated for compressive strength, slump height and cost of CRT concrete. Results show that incorporation of CRT as fine aggregate improves the compressive strength of concrete and this increase is directly proportional to its content. Authors therefore recommend up to 100% replacement of conventional fine aggregate with CRT in concrete production. The formulated models could predict compressive strength, slump and cost of CRT concrete if the mix ratio is known and vice versa. Analysis of variance and normal probability plots of model residuals were used to test adequacy of the models, and the models were found to be adequate at 95% confidence level. With the model equations, sample optimization was carried out to obtain the most economical mix for certain predefined criteria and the results were promising. Several similar optimizations can be carried out using the formulated model equations for any desired criteria of the modeled responses.

K. C. Nwachuckwu et al (2022) in this research work is aimed at using an optimization model based on Scheffe's Second Degree Polynomial (5,2) to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC).In this study, Scheffe's Second Degree Polynomial (5,2) model developed by Nwachukwu and others (2017) for five component mixture will be used to optimize the mix proportion that will produce the maximum strength of PFRC. Using Scheffe's Simplex method, the compressive strength of PFRC was determined for different mix ratios. Control experiments were also carried out and the compressive strength determined. After the tests have been conducted, the adequacy of the model was tested using Student's t-test. The test statistics found the model adequate for predicting the compressive strength of PFRC when the mix ratio is known. Optimum compressive strength for the Scheffe's(5,2) model was obtained as 25.23N/mm^2. Since structural concrete elements are generally made with concrete having a compressive strength of 20 to 35 MPa (or 20 to35N/mm^2), it then means that optimized PFRC based on Scheffe's model can produce the required compressive strength needed

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in major construction projects such as bridges and light-weight structures. Stakeholders in the construction industry are therefore advised to use the optimized PFRC, mainly for its economic and safety advantages.

METHODOLOGY

The analytical and experimental methods were used in this research work.

Analytical Method

The model is based on modified regression theory. A simplex lattice is described as a structural representation of lines joining the atoms of a mixture. The atoms are constituent components of the mixture. For a normal mortar mixture, the constituent elements are water, cement and fine aggregates (sand), which gives a simplex of a mixture of three components. Hence, the simplex lattice of this three-component mixture is an equilateral triangle.

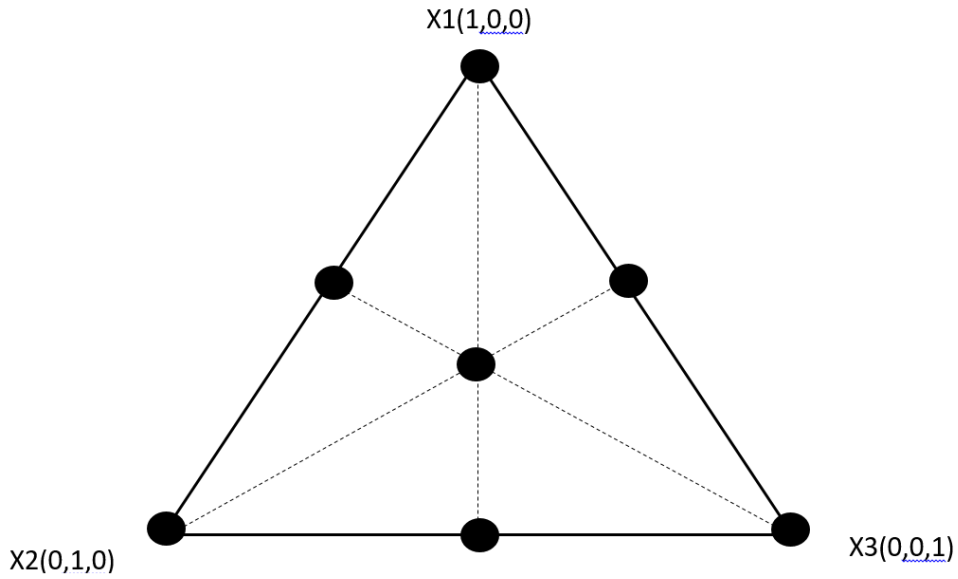


Fig. 2: Simplex-Centroid Design represented by an equilateral triangle for Three Variables.

Where;
X_1 = Water/Cement Ratio
X_2 = Cement
X_3 = Fine aggregate (Sand)
Therefore; $X1 + X2 + X3 = 1$

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$$\sum_{i}^{q} Xi = 1$$

Where q is the number component of a mixture and Xi is the proportion of the ith component in the mixture.

Experimental Method

The actual components as transformed from $X = BX$, becomes $Z = AX$. This is used to measure out the quantities of water (Z_1), cement (Z_2) and sand (Z_3) in their respective ratios for the compressive strength test. A total of ten mix ratios were used to produce 33 prototype mortar cubes measuring 150mm x 150mm x 150mm that were cured and tested on the 28th day. Three out of ten mix ratios were used a control mix ratios to produce 12 cubes for the confirmation of the adequacy of the mixture design model. The cubes were the crushed to determine its compressive strength using the universal testing machine. The load under which the specimen failed was recorded and used to compute the compressive strength of the cubes.

f = P/A

Where;

f = Compressive strength

P = Load under which the specimen failed

A = Area

RESULTS

Table 1: Results from Experimental and statistical design.

S/No	Pseudo-components			Response symbols	Replicates (N/mm²)			Average Response Y (N/mm²)	Real Components		
	X_1	X_2	X_3		1	2	3		Z_1	Z_2	Z_3
1	1	0	0	Y_1	16.2	19.4	16.85	17.50	0.400	1	2.60
2	0	1	0	Y_2	19.64	12.56	17.98	16.73	0.450	1	3.00
3	0	0	1	Y_3	15.34	16.67	15.54	15.85	0.500	1	3.40
4	½	1/2	0	Y_4	10.18	12.63	11.81	11.54	0.425	1	2.80
5	½	0	½	Y_5	14.83	9.64	14.82	13.10	0.450	1	3.00
6	0	1/2	½	Y_6	10.01	15.24	13.42	12.89	0.475	1	3.20
7	1/3	1/3	1/3	Y_7	16.57	14.83	14.22	15.21	0.450	1	3.00
Check points											
8	2/3	1/6	1/6	Y_8	17.68	16.09	12.63	15.47	0.425	1	2.80
9	1/6	2/3	1/6	Y_9	14.82	17.98	12.56	15.12	0.450	1	3.00
10	1/6	1/6	2/3	Y_{10}	15.44	13.42	11.41	13.42	0.475	1	3.20

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Discussion of Results

The research work "optimization of cement mortar compressive strength using the scheffe's lattice theory" was carried out in order to develop a statistical model from experimental data, to optimize the mix proportioning for mix ratio 1:3, to study the influence of mixture proportioning on its compressive strength. The quadratic and cubic scheffe's regression models were developed to meet up the aim and objectives of the study.

All data used were obtained by experimentation, i.e., casting mortar cubes corresponding to the various mix ratios obtained from the simplex, curing for the already stated period of time and then crushing in the universal crushing machine to obtain its compressive strength.

The following are some findings from the optimization of cement mortar compressive strength using the scheffe's simplex centroid.

Generally, the coefficient of determination (R^2) is a variable in regression models which shows the contribution of the independent to the estimated value of the dependent variable. R^2 value may also be used to measure the strength of the relationship of the variables, it does not follow the relationship as measured by R^2. The fact that two variables tend to increase or decrease together does not imply that one has a direct or indirect effect on the others. Both may be influenced by other variables in a manner that will give rise to strong mathematical relationship. This discussion should make it clear that the success with correlation coefficients requires familiarity with the field of application as well as their mathematical properties that both the reality and interpretation of R^2 depend heavily upon the extent to which the variables are joint normally distributed.

In the case of this model, og one dependent and three independent variables, the analysis of the data obtained in the optimization of cement mortar compressive strength using the scheffe's simplex centroid, showed a very strong relationship exist between compressive strength of mortar (Y) and the components of cement mortar i.e. Cement (Z_1), Sand (Z_2) and Water (Z_3).

The quadratic model was found to give a very strong relationship between compressive strength of mortar (Z_1) and the components of mortar i.e. Cement (Z_1), Sand (Z_2) and Water (Z_3). The R^2 value of 0.978 was obtained from the model, which indicates that about 97.8% of the dependent variable, mortar compressive strength (Y) is contributed by Cement (Z_1), Sand (Z_2) and Water (Z_3).

This model does not define compressive strength as a function of water (water to cement ratio, Z_3). This may not really yield the same result when the model is used at any randomly chosen water-cement ratio. Hence it is suggested that the model should be used in the water to cement ratio specified at the experimentation of this work i.e., 0.40 – 0.50.

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The special cubic model on the other hand, was found to give a very strong relationship between compressive strength of mortar strength (Y) and its components Cement (Z_1), Sand (Z_2) and Water (Z_3).

The R^2 value of 0.978 was obtained from the model which shows that about 97.8% of the dependent variable, mortar compressive strength (Y) is contributed by Cement (Z_1), Sand (Z_2) and Water (Z_3).

Hence the model defines the compressive strength of mortar as a function of all the three (3) components. The cubic model can be said to be more exact or appropriate in predicting cement mortar compressive strength, since all components are considered.

From the experimental and analytic optimization processes, the optimized compressive strength of mortar was found to be 17.05N/mm^2, corresponding to the mix proportion 1:2.6 at water-cement ratio 0.40, which is another basic objective of the work. The compressive strength of mortar was found to be optimized from 13.03N/mm^2, which is the compressive strength at control, to 17.05N/mm^2 which is the optimized compressive strength.

The optimized ratio only means that a lesser quantity of materials (sand, 2.6) can be used to achieve greater compressive strengths of cement mortar.

CONCLUSION

Two models were developed to optimize the compressive strength of cement mortar, that is; quadratic and special cubic models. From the analytic method of optimization employed in this work, it has been shown that the special cubic model gives a better result than the quadratic.

The correlation coefficient of both models were found to be very high, 0.978. This implies that a strong relationship exists between the compressive strength of cement mortar and its constituents. The P-value, level significance, was found to be 0.000 which is within acceptable limits since it is far less than 0.005

Recommendation

This work was carried out on cement mortar, mix ratio 1:3, it is therefore recommended that a more generalized model should be formulated, which can give an optimized compressive strength for a wider range of mix ratios. Also, other softwares like MINITAB, MATLAB, SAS, etc. may be used in the analysis of the work and the results may be compared to obtain the most effective.

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