E-ISSN: 1116 - 3240 P-ISSN: 3026-9334
MPRI-JAIMR | INTERNATIONAL JOURNAL OF AFRICAN INNOVATION & MULTIDSCIPLINARY RES.
VOL. 06 NO. 2 – OCTOBER, 2024

EVALUATING THE FORECAST PERFORMANCE OF ARMA-GARCH AND ST-GARCH USING NIGERIAN GROSS DOMESTIC PRODUCTS

[1]AKINTUNDE, MUTAIRU OYEWALE, [2]OLALUDE GBENGA ADELEKAN AND [2]OLAWALE, ABOLADE OLUDAYO

[1]Department of Statistics, Federal Polytechnic, Ayede, Oyo State. [2]Department of Statistics, Federal Polytechnic, Ede, Osun State.
Corresponding Author:
akintundemo@federalpolyayede.edu.ng

DOI Unique ID: 10.70382/mejaimr.v6i2.009

Abstract

Financial data must first be evaluated for forecast performance before being deemed appropriate for use in economic planning, according to policymakers, investors, academics, and end users. Failing to do so will undoubtedly result in a futile endeavor. The forecast accuracy of ARMA-GARCH and ST-GARCH is compared in this paper. Three SmoothTransition GARCH (ST-GARCH) models were examined: Logistic Smooth Transition (LST-GARCH), Exponential SmoothTransition (EST-GARCH), and Exponential Transition GARCH (ET-GARCH). After obtaining the relevant theoretical and mathematical frameworks, the gross domestic product of Nigeria was used as an empirical paradigm. Both the stationarity of the data and the descriptive statistics were looked at

Keywords: Gross domestic products, ARMA, GARCH, ET-GARCH, EST-GARCH, and LST-GARCH

EDITOR-IN-CHIEF
Prof. A.I. Mteiye [MALAWI]

in order to assess the state of the series used prior to its analysis.Here, the characteristics of the studied series demonstrate that it was stable at the first difference. It was then examined using specially written programs and the statistical analysis tool Statgraphics Centurion. The results showed that hybrids of LST-GARCH outperformed EST-GARCH, ARMA-GARCH, and ET-GARCH with exceptional performance. Thus, potential users are advised to utilize LST-GARCH.

Introduction

Forecast is part of life, any model that cannot give gainfully and fantastic results doesn't worth it as a result of this it is the dream of Econometrician and Time series analyst to get a very adequate and promising model that yield fantastic results. In the present study we shall look at the following model models ARMA, GARCH, ARMA-GARCH, ET-GARCH, LST-GARCH and EST-GARCH. The objective here is to look at all these models and determine which one yield the best forecast results.

LITERATURE REVIEW

In 2008, Mika Meitz and Pentti Saikkonen developed an equation for parameter estimation in nonlinear AR-GARCH models and found that the AR-GARCH model outperforms the standard AR and GARCH models by a significant margin.

Akintunde et.al. 2013 examined adequacy of GARCH models in forecasting exchange rates and concluded that GARCH models is not adequate, in other works Akintunde, et.al. (2013), Akintunde, et.al. (2021), Akintunde, (2021b) in their various works checked the hybrid models to examine Economic and Financial Time series and concluded that hybrid models outperformed traditional models of GARCH, ARMA, ARIMA and TAR models. Ghani and Rahim (2019) concluded that ARMA-GARCH outperformed classical GARCH model in Modeling and Forecasting of Volatility Malaysia Natural Rubber Prices. Nugroho, (2019), shows that all extended GARCH (1,1) models have the

potential to better fit than the standard GARCH(1,1) model. In the case of real application, the first and second best fitting specification are respectively provided by the GJRGARCH(1,1) and GARCH(1,1)-M models, meanwhile, the log-GARCH(1,1) model is outperformed by the GARCH(1,1) model.

GARCHNet is an excellent model that can explain conditional variance at least as well as conventional methods, as demonstrated by Mateusz Buczynski and Marcin Chlebus (2023). Value-at-risk projections are generally cautious, but as a result, fewer outliers are seen. Because of its comparatively greater opportunity cost, regulators would select GARCHNet more frequently than corporate management. We also see that this model has a significant advantage over GARCH models, which analyze fewer data samples. By acquiring significantly more data (see p values greater than 10), the model is able to make predictions of the same or superior quality.

THEORETICAL FRAMEWORK

The following models as used are as stated under

MODELS USED IN THE STUDY

AUTOREGRESSIVE MODEL

A Stochastic process $\{X_t\}$ is defined as

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_p X_{t-p} + e_t = \left(\phi_1 B + \phi_2 B^2 + ... + \phi_p B^p\right) X_t + e_t \quad (3.1)$$

using backward shift operator B, i.e $B^j X^k = X_{k-j}$ we can write (3.1) as

$\phi(B) X_t = e_t$ where $\phi(B) = 1 - \phi_1(B) - \phi_2(B^2) - ..... - \phi_p(B)^p$

ARMA

The $ARMA(p,q)$ model can be written as:

$$(1 - \varphi_1 B - \varphi_2 B^2 - ... - \varphi_p B^p) y_t = \theta_0 + (1 + \theta_1 B + \theta_2 B^2 + ... + \theta_q B^q)\varepsilon_t \quad (3.2)$$

where B is the backward shift operation, that is

$B^k Y_t = Y_{t-k}$ for k +ve integer

$\phi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - ... - \varphi_p B^p$

GARCH

The $GARCH\,(p,q)$ model is defined by:

$$y_t = \sigma_t \varepsilon_t \tag{3.3}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{3.4}$$

ET-GARCH

The model of exponential transition has the following form

$$G_A\left(\gamma, c; y_{t-j}\right) = \exp\left\{-\gamma y_{t-1}^2\right\}, \gamma > 0 \tag{3.5}$$

The function G_A Upon reaching zero, the function assumes symmetry and yields the number one. $G_A\left(\gamma, c; y_{t-j}\right) \to 0$ as $\left|\mathrm{y_{t\text{-}j}}\right| \to \infty$. The ET model's parameter vectors are devoid of intercept terms; hence, Equation (3.1) can be understood as a linear autoregressive model including time-varying coefficients that are stochastic. $\varphi + \theta G_A\left(\gamma, c; y_{t-j}\right)$.

EST-GARCH

The Exponential Smooth transition GARCH(EST-GARCH) model specification is

$$g_E = 1 - \exp\left\{-\gamma\left(y_t - c\right)^2\right\} \tag{3.6}$$

Therefore,

$$G_E = 1 - g_E = \exp\left\{-\gamma\left(y_t - c\right)^2\right\} = \exp\left\{-\gamma\left(\sigma_t \varepsilon_t - c\right)^2\right\} \tag{3.7}$$

LST-GARCH

This model is defined by Equation (6.13), where the transition function is the logistic function

$$g_L\left(\gamma, c; y_{t-j}\right) = \left\{1 + \exp\left\{-\gamma \prod_{i=1}^{k}\left(y_{t-j} - c\right)\right\}\right\}^{-1}, \gamma > 0 \tag{3.8}$$

EDITOR-IN-CHIEF
Prof. A.I. Mteiye [MALAWI]

The derivations used for the study were summarized in the table below as follows:

MODEL	GARCH	EXPONENTIAL TRANSITION-GARCH	EXPONENTIAL SMOOTH TRANSITION--GARCH	LOGISTIC SMOOTH TRANSITION--GARCH
MEAN	$\sigma_t \varepsilon_t$	$\frac{\gamma\alpha_0}{\sum_{j=1}^{p}(\alpha_i+\beta_j)-1}$	$\gamma\left[\frac{\alpha_0}{\sum_{j=1}^{p}(\alpha_i+\beta_j)-1}+c^2\right]$	$-\gamma c^2$
VARIANCE	$\frac{\alpha_0}{1-\sum_{i=1}^{p}(\alpha_i+\beta_j)}$	$\gamma^2\left\{\frac{2\sigma^4}{n-1}\right\}$	$=2\gamma^2\sigma^2\left[\frac{\sigma^2+2c^2\sqrt{n-1}\sqrt{2}}{n-1}\right]$	$=2\gamma^2\sigma^2\sqrt{\frac{2}{n-1}}$

MODELS SELECTION AND EVALUATION

The following models were used for models selection and evaluation

MODELS ADEQUACY SELECTION

i. $AIC=-2Ln(L)+2k$

ii. $AICC=AIC+2k\frac{(k+1)}{N-K-1}$

iii. $SBC=-2Ln(L)+Ln(N)k$

iv. $HQC=-2Ln(L)+2Ln(Ln(N))k$

MODELS PERFORMANCE MEASURE INDICES

i. $MSE=\frac{1}{N}\sum_{n=1}^{N}\left(\sigma_n^2-\hat{\sigma}_n^2\right)^2$

ii. $RMSE=\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\sigma_n^2-\hat{\sigma}_n^2\right)^2}$

iii. $MAPE=\frac{1}{N}\sum_{n=1}^{N}\left|\left(\sigma_n^2-\hat{\sigma}_n^2\right)^2 X100\right|$

EDITOR-IN-CHIEF
Prof. A.I. Mteiye [MALAWI]

DATA ANALYSIS AND INTERPRETATION OF RESULTS

Unit root test output for the level for the series

At this stage, an Augmented Dickey-Fuller (ADF) unit root test was performed on the series that was being studied. They were chaotic, violent, and extremely noisy at level (original data); nevertheless, as tables 1 and 2 below demonstrate, they became stationary at first difference.

Table 1: Unit root test output for original data

Series	ADF-Test statistic	Critical value (5%)	Mackinnon prob
NIG. GDP	-1.6505	-1.6899	0,0528

Table 2: Unit root test output for the first difference of the series

Series	ADF-Test statistic	Critical value (5%)	Mackinnon prob
NIG. GDP	-2.3218	-2.8729	0.0000

Table 3: Analysis of Autocorrelation function of Series under consideration (GDP)

Lag	*Autocorrelation*	*Standard Error*	*Lower Prob. Limit (95.0%)*	*Upper Prob. Limit (95.0%)*
1	0.48453	0.152499	-0.298892	0.298892
2	0.437058	0.184866	-0.362331	0.362331
3	0.146056	0.207509	-0.406711	0.406711
4	0.0712786	0.209886	-0.41137	0.41137
5	0.097359	0.210448	-0.412472	0.412472
6	0.00357834	0.211493	-0.41452	0.41452
7	-0.129795	0.211495	-0.414522	0.414522
8	-0.0281639	0.213339	-0.418137	0.418137
9	-0.180571	0.213425	-0.418307	0.418307
10	0.0725969	0.216949	-0.425213	0.425213
11	0.0310927	0.217513	-0.426319	0.426319

12	0.196544	0.217617	-0.426522	0.426522
13	0.0280429	0.221706	-0.434538	0.434538
14	-0.0152156	0.221789	-0.434699	0.434699

Source: output of analyzed data 2024

Interpretation

The estimated autocorrelations between GDP statistics at various lags are shown in Table 3 above. The lag k autocorrelation coefficient calculates the GDP correlation between time and value. The 95% probability boundaries around 0 are also shown. If the computed coefficient is excluded from the probability bounds for a specific latency, then there is a statistically significant link at the 95% confidence level. In this case, two of the 24 autocorrelation coefficients are statistically significant at the 95% confidence level, suggesting that the time series may not be completely random (white noise). The series' autocorrelation behavior is depicted in the image below.

Table 4: Estimated Partial Autocorrelations Nigerian GDP

Lag	*Partial Autocorrelation*	*Standard. Error*	*Lower Prob. Limit (95.0%)*	*Upper Prob. Limit (95.0%)*
1	0.48453	0.152499	-0.298892	0.298892
2	0.264349	0.152499	-0.298892	0.298892
3	-0.193629	0.152499	-0.298892	0.298892
4	-0.0585925	0.152499	-0.298892	0.298892
5	0.175984	0.152499	-0.298892	0.298892
6	-0.0885899	0.152499	-0.298892	0.298892
7	-0.282484	0.152499	-0.298892	0.298892
8	0.22848	0.152499	-0.298892	0.298892
9	-0.111092	0.152499	-0.298892	0.298892
10	0.1347	0.152499	-0.298892	0.298892
11	0.081847	0.152499	-0.298892	0.298892

12	0.170245	0.152499	-0.298892	0.298892
13	-0.325221	0.152499	-0.298892	0.298892
14	-0.0599901	0.152499	-0.298892	0.298892

Table 4 above shows the predicted partial autocorrelations between the GDP statistic at various lags. The lag k partial autocorrelation coefficient, which accounts for correlations at all lower lags, calculates the GDP correlation between values for different times. Which autoregressive model order is necessary to match the data can be ascertained using it. Additionally, 95% probability boundaries around 0 are shown. If the expected coefficient is absent from the probability bounds for a given lag, then there is a statistically significant relationship at that lag at the 95% confidence level. In this case, two of the twenty-four partial autocorrelation coefficients are statistically significant at the 95% confidence level.

Table 5: Periodogram for Gross Domestic Product

I	*Frequency*	*Period*	*Ordinate*	*Cumulative Sum*	*Integrated Periodogram*
0	0.0		4.85241E-30	4.85241E-30	4.17776E-33
1	0.0232558	43.0	361.247	361.247	0.311021
2	0.0465116	21.5	65.9654	427.213	0.367815
3	0.0697674	14.3333	134.023	561.236	0.483205
4	0.0930233	10.75	102.148	663.384	0.571151
5	0.116279	8.6	65.3243	728.708	0.627393
6	0.139535	7.16667	7.06967	735.778	0.633479
7	0.162791	6.14286	84.6839	820.462	0.706389
8	0.186047	5.375	50.3015	870.763	0.749697
9	0.209302	4.77778	17.6469	888.41	0.76489
10	0.232558	4.3	43.9898	932.4	0.802764

11	0.255814	3.90909	5.11242	937.512	0.807166
12	0.27907	3.58333	1.09654	938.609	0.80811
13	0.302326	3.30769	2.73606	941.345	0.810465
14	0.325581	3.07143	0.186337	941.531	0.810626
15	0.348837	2.86667	23.6416	965.173	0.83098
16	0.372093	2.6875	17.9832	983.156	0.846463
17	0.395349	2.52941	46.3167	1029.47	0.88634
18	0.418605	2.38889	43.6186	1073.09	0.923895
19	0.44186	2.26316	0.732964	1073.82	0.924526
20	0.465116	2.15	1.46141	1075.29	0.925784
21	0.488372	2.04762	86.201	1161.49	1.0

Interpretation

Table 5 displays the GDP periodogram ordinates. It is frequently used to identify data cycles with consistent frequencies. A sequence of sine functions is fitted at each of the 22 frequencies to create the periodogram. The squared amplitudes of the sine functions are the ordinates. Since the total corrected sum of squares in an ANOVA table equals the sum of the ordinates, the periodogram can display analysis of variance by frequency.

Table 6: Variances of ST-GARCH models with ARMA, GARCH and ARMA-GARCH model

SERIES	ARMA	GARCH	ARMA-GARCH	ET-GARCH	EST-GARCH	LST-GARCH
GDP	19.2311	18.9321	18.7321	18.1243	16.5687	16.0012

Table 6 above shows that the ST-GARCH (ET-GARCH, EST-GARCH, and LST-GARCH) model produced the least variance, followed by ARMA-GARCH, GARCH, and ARMA in that order. Nevertheless, LST-GARCH produced the greatest outcomes within the ST-GARCH committee.

TABLE 6: MODELS VERSUS PERFOMANCE MEASURE INDICES

MODEL	GARCH	ARMA	ARMA-GARCH	ET-GARCH	EST-GARCH	LST-GARCH
RMSE	1.3216	1.1124	1.0125	0.9972	0.9787	0.8971*
MAE	2.1231	2.1001	2.0019	1.8976	1.8643	1.6532*
MAPE	13.2764	12.9872	11.9765	10.0973	9.8723	7.0997*

For the three performance measure indices (RMSE, MAE, and MAPE), the result shows that LST-GARCH performed remarkably well, with EST-GARCH closely lagging behind. In the absence of LST-GARCH, this has the policy effect of permitting the use of EST-GARCH.

TABLE 7: MODELS VERSUS ADEQUACY TESTS MEASUREMENT INDICES

MODEL	GARCH	ARMA	ARMA-GARCH	ET-GARCH	EST-GARCH	LST-GARCH
AIC	124.7460	119.0217	112.2317	98.2819	98.1023	96.0019
AICC	124.7321	117.2311	111.2619	102.8219	102.3211	100.2317
SBC	124.,7467	118.7231	116.9299	115.0211	114.2918	111.2918
HQC	129.1121	112.211	111.0241	114.2198	98.9991	98.3219

Model definition comes after determining the residuals' heteroscedasticity. Six models are produced, each derived from distinct conditional variance and conditional mean specifications in the different models that are employed. To evaluate each of the six distinct specification models and identify the best one, we employed metrics such as AIC, AICC, SBC, and HQC. Table 7 displays the six models in comparison. The minimal values of AIC, AICC, SBC, and HQC are displayed by both the LST-GARCH and the EST-GARCH models, indicating their close relationship.

SUMMARY OF MAJOR FINDINGS AND CONCLUSIONS

The study started by examining the descriptive conditions and statonarity of the series in question (Nigerian Gross Domestic Products); all of the series'

characteristics were found to be stable at first difference. The lag k partial autocorrelation coefficient, which accounts for correlations at all lower lags, calculates the GDP correlation between values at various dates. Additionally, 95% probability boundaries around 0 are shown. There is a statistically significant relationship at a specific latency at the 95% confidence level. In this case, two of the twenty-four partial autocorrelation coefficients are statistically significant at the 95% confidence level.

The periodogram is made by fitting a series of sine functions at each of the 22 frequencies. The ordinates are equal to the sine functions' squared amplitudes. At each of the 22 frequencies, a sequence of sine functions is fitted to create the periodogram. The ordinates are the squared amplitudes of the sine functions.

Table 7 above shows that the variance analysis showed that the ST-GARCH (ET-GARCH, EST-GARCH, and LST-GARCH) model produced the least variance, followed by ARMA-GARCH, GARCH, and ARMA in that order. Nevertheless, LST-GARCH produced the greatest outcomes within the ST-GARCH committee. The models' performance against the performance measure indices (RMSE, MAE, and MAPE) shows that LST-GARCH fared better than EST-GARCH, but it trailed it by a small margin. The policy consequence of this is to allow the employment of EST-GARCH in the absence of LST-GARCH.

To identify the optimum specification model, each of the four models was evaluated using models versus adequacy measure indices such as AIC, AICC, SBC, and HQC. When the four models are compared, the LST-GARCH model performs the best.

In conclusion, LST-GARCH is advised for potential users. But if this isn't available, we can use EST-GARCH models.

Financial support and sponsorship

This study was sponsored by Tertiary Education Trust Fund (2022-2024 TETfund) merged Intervention.

References

Akintunde, M.O. (2021) On evaluating the volatility of Nigerian gross domestic product using Smooth Transition Autoregressive GARCH (STAR -GARCH) models. American Journal of Applied Mathematics and Statistics, 2021, Vol. 9, No. 3, 102-106.

Akintunde, M.O. (2021) On forecasting malaria infection using autoregressive integrated moving average (ARIMA) models. Further Applied Mathematics, 1 (2) (2021): 1-10.

Akintunde, M.O., Kgosi, P.M. and Shangodoyin, D.K. (2013), Evaluation of GARCH model adequacy in forecasting non-linear economic time series, Journal of Computation and Modeling, 3(2):1-20.

Akintunde, M.O., Shangodoyin, D.K. and Kgosi, P.M. (2013), Smooth Transition Autoregressive-GARCH model in forecasting non-linear economic time series data, Journal of Statistical and Econometric Methods, 2(2):11-19

Akintunde, M.O., Kgosi, P.M. and Shangodoyin, D.K. (2013), Smooth Transition GARCH model in forecasting non-linear economic time series data, Progress in Applied Mathematics, 5(2):30-39

Akintunde, M.O., Shangodoyin, D.K. and Kgosi, P.M. (2013), Measuring the forecast performance of GARCH and Bilinear-GARCH models in time series data, American Journal of Applied Mathematics, 1(1):17-23

Ghani and H A Rahim 2019 ."Modeling and Forecasting of Volatility using ARMA-GARCH: Case Study on Malaysia Natural Rubber Prices", IOP Conf. Ser.: Mater. Sci. Eng. 548 012023 DOI 10.1088/1757-899X/548/1/012023.

Mika Meitz & Pentti Saikkonen, 2008. "Stability of nonlinear AR-GARCH models," Journal of Time Series Analysis, Wiley Blackwell, vol. 29(3), pages 453-475, May.

Nugroho D B, Susanto B, Prasetia K N P and Rorimpandey R 2019 Jurnal Akuntansi dan Keuangan 21 12

EDITOR-IN-CHIEF
Prof. A.I. Mteiye [MALAWI]