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MHD CASSON FLUID FLOW OVER A LINEAR STRETCHING SHEET WITH THE EFFECT OF VISCOUS DISSIPATION

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Abstract

Casson fluid is a type of non-Newtonian fluid also known as shear-thinning fluid whose viscosity decreases with increased shear rate. This study examines the flow of Casson fluid over a stretching sheet, investigating the effects of Casson, magnetic, porosity, and viscous dissipation parameters. The inclusion of viscous dissipation serves as the novelty of this study, as it significantly impacts the thermal behavior of the fluid, particularly in high-velocity flows or fluids with high viscosity. The governing nonlinear partial differential equations are transformed into ordinary differential equations using similarity transformations and solved using shooting techniques embedded in MAPLE 17. The results indicate that the velocity profile decrease with increasing Casson parameter, magnetic parameter and porosity parameter while the temperature profile increase with increasing Eckert number, Casson parameter, magnetic parameter and porosity parameter but decline with increase in Prandtl number.

Introduction

Magnetohydrodynamic (MHD) Casson fluids are important in many industrial and biomedical applications because of their complex and nonlinear behavior, which can be tailored to achieve specific desired properties. Casson fluids are characterized by a nonlinear relationship between the shear stress and the shear rate, which means that their viscosity changes depending on the intensity of the applied force

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(Xuan and Shab, 2024). The study of real-life problems dealing with flow models of non-Newtonian fluids has received a special attention due to their several possible applications in engineering and industries (Mohammed *et. al*, 2020).**The study of laminar flow and heat transfer of a viscous fluid over a stretching sheet is an essential research field in fluid mechanics, due to its extensive applications in many manufacturing processes in industry, such as glass-fiber production, extraction of polymer sheet, hot rolling, wire drawing, solidification of liquid crystals, paper production, drawing of plastic films, petroleum production, exotic lubricants and suspension solutions, continuous cooling and fibers spinning (**Saidulu and Lakshmi, 2016). **Casson fluid is another type of non-Newtonian that exhibits yield stress. Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear (Dash et al., 1996). The examples of Casson fluids include substances like jelly, human blood, honey, soup, tomato sauce and concentrated fruit juices.**

The flow of various non-Newtonian fluids over stretching sheets was analyzed by Emmanuel *et al*., 2015, Wahidzzaman *et al*., 2014, Sharada and Shnkar, 2016, Moreau, 1990, Asogwa and Ibe, 2020, Makanda *et al.,* 2015, Saidulu and Lakshmi, 2016.

Mohammed *et.al* (2025) examined two dimensional Casson fluid flow over a stretching surface with thermal and solutal buoyancy effect. Pushapalata *et al*. (2017) analyzed the effects of cross diffusion on casson fluid over an unsteady stretching surface with boundary effects. Mohammed and Mohammed (2025) analyzed MHD Casson fluid flow over a permeable stretching sheet. Charankumar *et al.* (2016) examined chemical reaction and Soret effects on casson MHD fluid flow over a vertical plate with heat source/sink. Mohammed *et.al* (2025) studied the effect of Casson fluid flow with heat and mass transfer over an exponential stretching Sheet. Saidulu and Lakshmi (2016) described the boundary layer flow of non-Newtonian Casson fluid accompanied by heat and mass transfer towards a porous exponentially stretching sheet with velocity slip and thermal slip conditions in presence of thermal radiation, suction/blowing, viscous dissipation, heat source/sink and chemical reaction effects. Saidu *et al*. (2025) worked on the Mathematical study of Casson fluid flow over a stretching surface with heat generation effect. Mohammed *et.al* (2020) examined the effect of radiation on MHD Casson fluid flow over an exponential stretching Sheet. (Xuan and Shab, 2024) Investigated the MHD flow of a Casson fluid through a porous media stretching sheet numerically, ignoring the effect of viscous dissipation. This study seeks to extend their study by incorporating viscous dissipation, as it significantly impacts the thermal behavior of the fluid, particularly in high-velocity flows or fluids with high viscosity.

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Model Formulation

Considering the steady two-dimensional MHD laminar flow with heat transfer of an incompressible Casson fluid flow past a porous stretching sheet with stretching velocity $u_w = ax$. The Cartesian coordinate system is chosen such that y - axis is along the vertical direction and x - axis normal to the plate and application of magnetic field as B_0. Given that u and v are the velocity component in x and y direction, T_w is temperature at the wall of the sheet, a is constant and T_∞ is ambient condition.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is given as in (Prakash *et al*., 2016; Saidu *et al*., 2021; Ramachandra *et al*. 2013; Kumar and Gangadhar, 2015; Rao and Sreenadh, 2017, Mohammed *et al*., 2020 and Mohammed *et al*., 2025) is given by:

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}}\right)e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, \pi < \pi_c \end{cases} \tag{1}$$

Where $\pi = e_{ij}e_{ij}$ and e_{ij} denotes the $(i, j)^{th}$ components of the deformation rate, π is the product of the deformation rate itself, π_c is the critical value of this product based on the non-Newtonian fluid model, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid and p_y is the yield stress of the fluid.

The governing boundary layer equations for the steady incompressible casson fluids flow are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\upsilon}{K^*}u - \frac{\sigma {B_0}^2}{\rho}u \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \left(1 + \frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 \tag{4}$$

Subject to the boundary conditions

$$\left.\begin{array}{l} u = u_w, v = 0, T = T_w at\ y = 0 \\ u \to 0, T \to T_\infty as\ \ y \to \infty \end{array}\right\} \tag{5}$$

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Where u and v are the velocity components in the direction of x and y respectively, β is the Casson fluid parameter, υ is the kinematic viscosity, B_0 is the strength magnetic field, K^* is the permeability of porous medium, T is the temperature, T_w is the temperature at the wall, T_∞ is the free stream temperature, α is the thermal diffusivity, ρ is the density of the fluid, g is acceleration due to gravity.

Method of Solution

To convert the governing partial differential equations into a nonlinear and coupled ordinary differential equations using similarity transformations.

The following transformation is defined.

$$\left.\eta = \sqrt{\frac{a}{\upsilon}}y,\ \psi = \sqrt{a\upsilon}xf(\eta) \text{and } \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\right\} \tag{6}$$

η as Dimensionless coordinate, ψ as Stream function and $\theta(\eta)$ as Dimensionless Temperature.

Using the definition of stream function, $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$

The transformed equations and their boundary conditions are:

$$\left(1 + \frac{1}{\beta}\right) f'''(\eta) + f(\eta) f''(\eta) - \left(f'(\eta)\right)^2 - \left(M + K\right) f'(\eta) = 0$$
$$f(0) = 0, \quad f'(0) = 1, \ f'(\eta \to \infty) \to 0 \tag{7}$$

$$\frac{1}{P_r}\theta''(\eta) + f(\eta)\theta'(\eta) + E_c\left(1 + \frac{1}{\beta}\right)\left(f''(\eta)\right)^2 = 0$$
$$\theta(0) = 1, \quad \theta(\eta \to \infty) \to 0 \tag{8}$$

Where

$$\left. M = \frac{\sigma B_0^{\ 2}}{a\rho},\ K = \frac{\upsilon}{aK^*},\ E_c = \frac{u_w^{\ *}}{c_p\left(T_w - T_\infty\right)},\quad P_r = \frac{\upsilon}{\alpha}\right\} \tag{9}$$

Results and Discussion

The governing partial differential equations were reduced to ordinary differential equations using similarity transformation and solved using shooting technique embedded in MAPLE 17 package. The graphical illustrations for velocity and temperature profiles

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were discussed for different physical parameters which includes, Casson parameter β, Prandtl number P_r, permeability parameter K and magnetic parameter M as shown below.

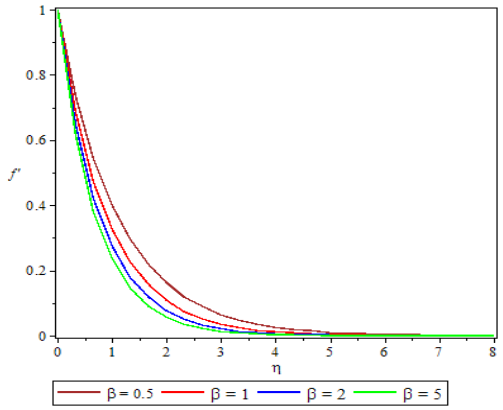


Figure 1: Influence of β on Velocity Profile

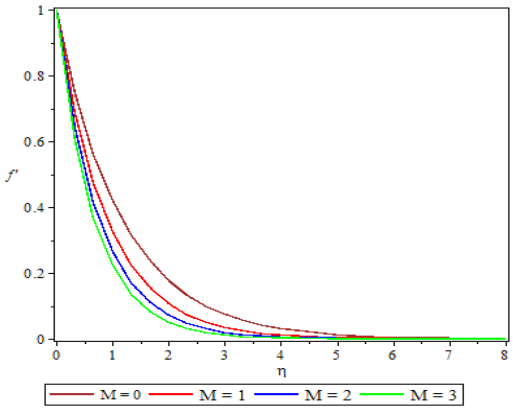


Figure 2: Influence of M on Velocity Profile

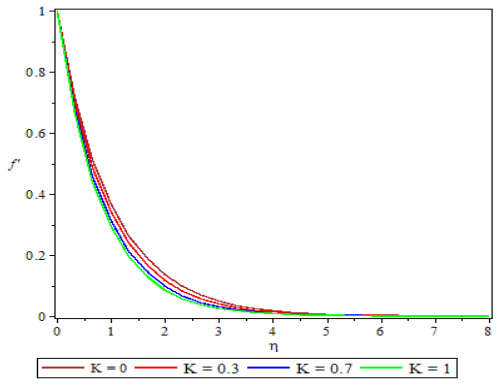


Figure 3: Influence of K on Velocity Profile

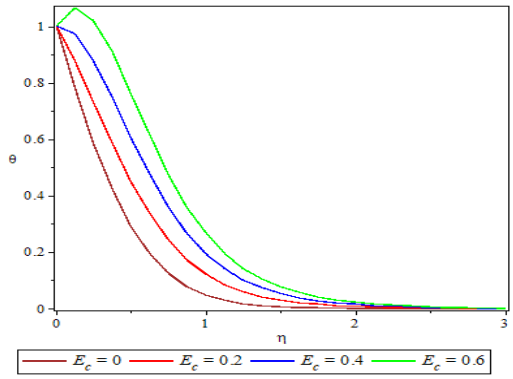


Figure 4: Influence of E_c on Temperature Profile

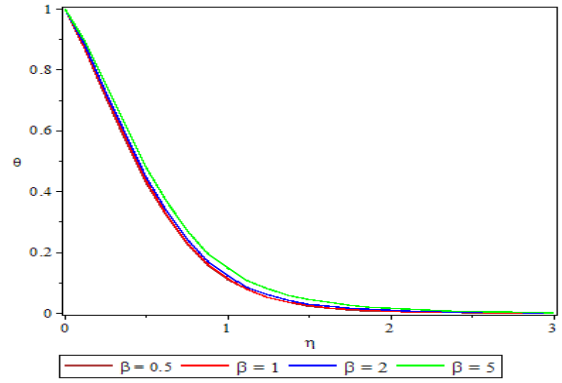


Figure 5: Influence of β on Temperature Profile

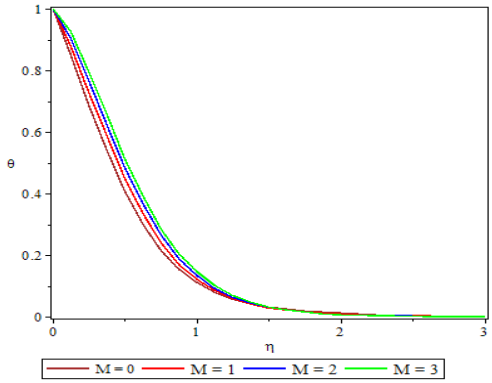


Figure 6: Influence of M on Temperature Profile

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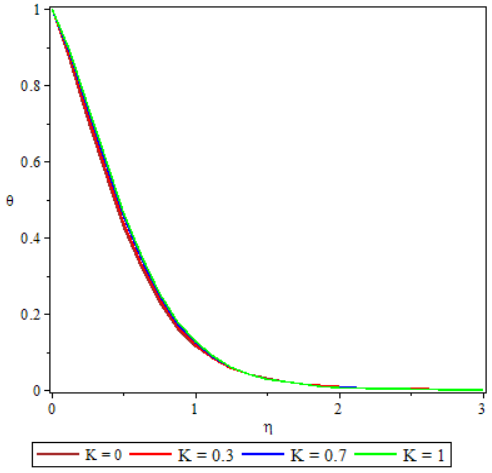


Figure 7: Influence of K on Temperature Profile

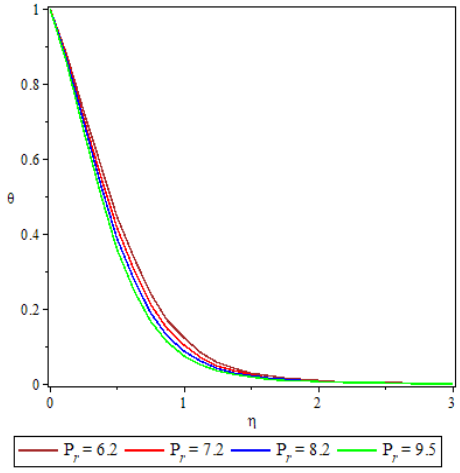


Figure 8: Influence of P_r on Temperature Profile

Figures 1 to 3 depicts the graph of velocity profiles against the similarity variable η for different values of β, M and K, it was observed from these figures that, as β, M and K increased, the fluid velocity distribution decreased inside the boundary layer, that is velocity profile decrease with increasing Casson parameter, magnetic parameter and porosity parameter. The temperature profile increase with increasing Eckert number, Casson parameter, magnetic parameter and porosity parameter as shown in figures 4,5,6 and 7, but declined with increase in Prandtl number as revealed in figure 8.

Conclusion

The study reveals that:

- Increase in Casson parameter, magnetic parameter and porosity parameter decrease the fluid flow velocity and the boundary layer thickness, these indicates an increase in total resistive force within the boundary layer.
- Increase in Eckert number, Casson parameter, magnetic parameter and porosity parameter enhance the temperature profile but declined with increase in Prandtl number. The enhancement of the temperature profile by Eckert number, Casson parameter, magnetic parameter and porosity parameter suggest that these factors generate heat while the decline in temperature profile due to increase in Prandtl number suggest that the fluid has lower thermal conductivity relative to its viscosity leading to slow diffusion of heat from the sheet to the fluid and ultimately leading to a decrease in temperature profile.

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- The Casson, magnetic and porosity parameters serve as effective tools for controlling fluid flow velocity and its stability, leading to thermal accumulation within the thermal boundary layer. For industrial processes such as polymer extrusion or metal spinning, this suggest that applying a magnetic field to stabilize the fluid flow must be balanced with higher Prandtl number for enhanced cooling to prevent the material from overheating due to magnetic and viscous dissipation.

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