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# OPTIMIZATION OF SERVICE EFFICIENCY USING AN OPEN JACKSON QUEUING NETWORK MODEL IN THE NIGERIAN BANKING SECTOR

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## Abstract

Many banks in Nigeria experience persistent challenges related to long waiting times, congestion at service points, and inefficient allocation of service personnel. These issues often arise due to an imbalance between customer arrival rates and service capacity across different service units. This study applies the Open Jackson queuing network model to evaluate and optimize customer service efficiency in First Bank Plc., Bida. Data were collected over a four-week period through direct observation and interviews, capturing arrival rates, service rates, and routing probabilities among service nodes. The banking system was modeled as a multi-node queuing network, where each department represents a service node. Performance measures including traffic intensity, queue length, and waiting time were computed. Results indicate that the existing system experiences significant congestion at key service nodes, leading to high waiting times. A modified model incorporating optimal server allocation was developed. The findings show that the total expected waiting time of the customer in the system reduced from 58 minutes to 49 minutes after optimization. The study concludes that optimal server allocation significantly improves service efficiency and customer satisfaction.

**Keywords:** Queuing theory, Open Jackson network, Service efficiency, node, Server.



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# Introduction

Service efficiency has become a critical concern in modern banking systems, where customers expect fast and reliable service delivery. In Nigeria, banks continue to face challenges such as long waiting times, congestion, and inefficient allocation of service personnel. These issues reduce operational efficiency and negatively impact customer satisfaction, managing such systems without analytical tools often results in inefficiencies.

Queuing theory provides a framework for analyzing such systems by evaluating performance measures like arrival rates, service rates, and waiting times. Recent studies have shown that proper server allocation improves system performance (Adeleke & Adebiyi, 2021; Gupta & Singh, 2022; Olanrewaju et al., 2023). However, many Nigerian banks still rely on intuition rather than data-driven models.

In a typical banking environment, customers do not receive all services at one point. Instead, they move from one service unit to another, such as customer advisory, customer service, marketing, and teller units. This creates a network of queues where congestion at one point can affect the entire system. Managing such a system manually or based only on experience is often inefficient and may lead to poor service delivery. The Open Jackson queuing network model is a powerful mathematical tool that can be used to analyze such systems. It allows customers to enter the system, move between different service points, and exit after receiving service. This model helps to evaluate important performance measures such as arrival rate, service rate, queue length, and waiting time. With this information, it becomes possible to identify problem areas and improve the system.

Several studies have applied queuing theory to improve service efficiency. For example, research in hospital systems has shown that increasing the number of servers at critical points can significantly reduce waiting time and improve service quality. Similar results have been obtained in banking and other service industries, where proper allocation of resources leads to better performance and customer satisfaction. (Owoloko, et al., 2025) and (Emmanuel and Lawal, 2020).

Despite these advancements, many banking systems, especially in developing regions, still experience long queues and delays. This is often due to poor planning and lack of proper analytical tools for decision-making. In First Bank Plc., Bida, customers pass through multiple service units, and delays at some of these units suggest that the current system may not be operating efficiently. Therefore, there is a need to carefully study the existing system and determine how it can be improved. This study applies the Open Jackson queuing network model to analyse and optimize service efficiency at First Bank Plc., Bida.



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**Problem Statement**

Despite having multiple service units, banks still experience long waiting times due to poor server allocation and lack of analytical optimization tools. This results in congestion and reduced customer satisfaction. Therefore, there is a need to apply a mathematical model to improve system efficiency.

**Objectives of the Study**

The main objective is to optimize service efficiency using a queuing network model.
The specific objectives are:

- To capture the queuing system of First Bank, Plc., Bida as network of queue.
- To determine the arrival rate of customers into each of the node.
- To determine the departure rate of customers from each of the node
- To determine the waiting time of a customers in the system.
- To determine the optimal number of servers at each node in order to minimize time spent by the customer in the system.
- To compare existing and improved systems.

**Research Questions**

- What are the arrival and service rates of the customers at each node?
- How efficient is the current systems?
- Which nodes experience congestion?
- What is the optimal number of servers?
- How does the optimized system perform?

**Literature Review**

Queuing theory originated from the work of Agner Krarup Erlang in the early $20^{th}$ century, where he analyzed congestion in telephone traffic systems. Since then, it has evolved into a major branch of operations research with wide applications in telecommunications, healthcare, transportation, manufacturing, and service industries. The fundamental objective of queuing theory is to model and analyze systems in which customers compete for limited service resources, with the aim of optimizing performance measures such as waiting time and system utilization.

A queuing system typically consists of an arrival process, a service mechanism, and a queue discipline. In more complex environments, such as banking systems, customers do not receive all services at a single point but instead move through multiple interconnected service nodes. This gives rise to queuing networks, where the output of one node



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becomes the input to another. Such systems are more accurately represented using network-based models rather than single-node queuing models.

The Open Jackson queuing network is one of the most widely used models for analyzing multi-node service systems. It assumes that arrivals follow a Poisson process, service times are exponentially distributed, and customer routing between nodes is governed by fixed probabilities. These assumptions allow for tractable analysis and the derivation of steady-state performance measures across the network. According to Zhang et al. (2021), Open Jackson networks are particularly effective in identifying bottlenecks and evaluating system-wide performance in service-oriented environments.

Several empirical studies have applied queuing models to improve service efficiency. In the health sector, Lawal et al. (2019) analyzed a multi-server queuing system at a National Health Insurance Scheme unit and found that optimal server allocation significantly reduced waiting and service costs. The two conflicting costs were balanced and the optimal performance for the queuing system was then determined. It was concluded that for the morning session, the average queuing length, waiting time of Customer as well as over utilization of doctors at the unit could be reduced at an optimal sever level of 3 doctors at a minimum total cost of ₦6219.98 per hour as against the present server level of 2 doctors with high total cost of ₦26025.12 per hour which include waiting and service costs and for the evening session the present sever level of 1 doctor should be maintained. The researchers recommend that the result of the research is important information to the management of NHIS unit of the General Hospital Minna to provide better service to the Customer at a minimum cost.

More recently, Olanrewaju et al. (2023) examined Nigerian banking systems and reported that data-driven queuing analysis improves decision-making and enhances customer satisfaction.

Similarly, Lawal et al.(2019) and Olanrewaju et al. (2023) confirm that queuing models improve service delivery. However, limited research exists on multi-node banking systems using Open Jackson networks. A successful application of queuing network system in hospital management was done previously by Owoloko, et al., 2015 and Emmanuel & Lawal, 2020.

Kembe et al. (2012) demonstrated that increasing the number of service channels in hospitals systems leads to improved patient flow and reduced congestion.

In the Banking Sector, Adeleke and Adebiyi (2021) applied queuing models to evaluate customer service operations and concluded that inefficient server distribution is a major cause of long waiting times. Gupta and Singh (2022) further showed that balancing service capacity with customer arrival rates is essential for achieving optimal system performance.



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Despite the growing body of literature, many existing studies focus on single-node systems or do not adequately capture the complexity of customer movement across multiple service units. Furthermore, there is limited research on the application of Open Jackson queuing networks in Nigeria banking environments. Particularly at the branch level. This creates a gap in the literature that necessitates further investigation.

This study addresses this gap by applying an Open Jackson queuing network model to a real banking system in Nigeria. Unlike previous studies, it considers the interdependence of multiple service nodes and evaluates system performance before and after optimization. By doing so, it contributes to both the theoretical development and practical application of queuing models in service systems.

**Methodology**

The type of queuing system adopted by organizations depends solely on the type of service being provided. The First Bank Plc. Bida practices a network-type of queuing system. A queuing network is composed of several random queue systems, mostly limited and single queue systems. Diverse types of customer move through the network in many ways and are served by the service nodes within the network system. A queuing network system has a set of nodes (*i*). Each node has a number of servers (*s*) and a single node can be regarded as a queuing system.

**Collection of Data**

The First Bank Plc. is located along Esso main road and is made up of five different departments. In this study, each department is regarded as node of the network system. The data used in this research were gathered from the five departments of the First Bank Plc. and were obtained based on the arrival and departure rate as well as time spent at each node. The method of data gathering was direct observation and personal interview by the research team. The data gathering was for a period of four weeks. This data gathering was carried out from Monday to Friday. In a day, gathering of the data was for a total of six (6) hours at different times of the day. For each node, the number of arrivals and departures together with service times were taken at intervals of 4-minute arrivals for customer arrivals into a node, while the departure rate was obtained also by the average number of four (5) minutes departures of customer at that particular node. Each of the nodes was observed for a period of one (1) hour daily. All the nodes under consideration have at least a server.

***Model Assumptions***

The following assumptions were made in modeling the system:



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- The First Bank Plc. Bida network queuing system is considered as an independent queuing system.
- Each node in the Bank has at least a server with exponential services time.
- Queuing discipline in Bank is first come first served.
- The external arrival pattern follows a Poisson arrival process.
- The service rate in the Bank depends on the number of Customer at each node of the system.
- In the bank, the bank personnel's are regarded as servers.

We consider a banking network queuing system based on open Jackson network queuing model, the First Bank. Plc. constitute of five units. In this study, we assumed that customers who come in to bank for services will start by going first to the customer's advisory unit to see and then move to the customer services unit, from their customers proceed to see teller unit. This procedure customer until the customer depart from the bank.

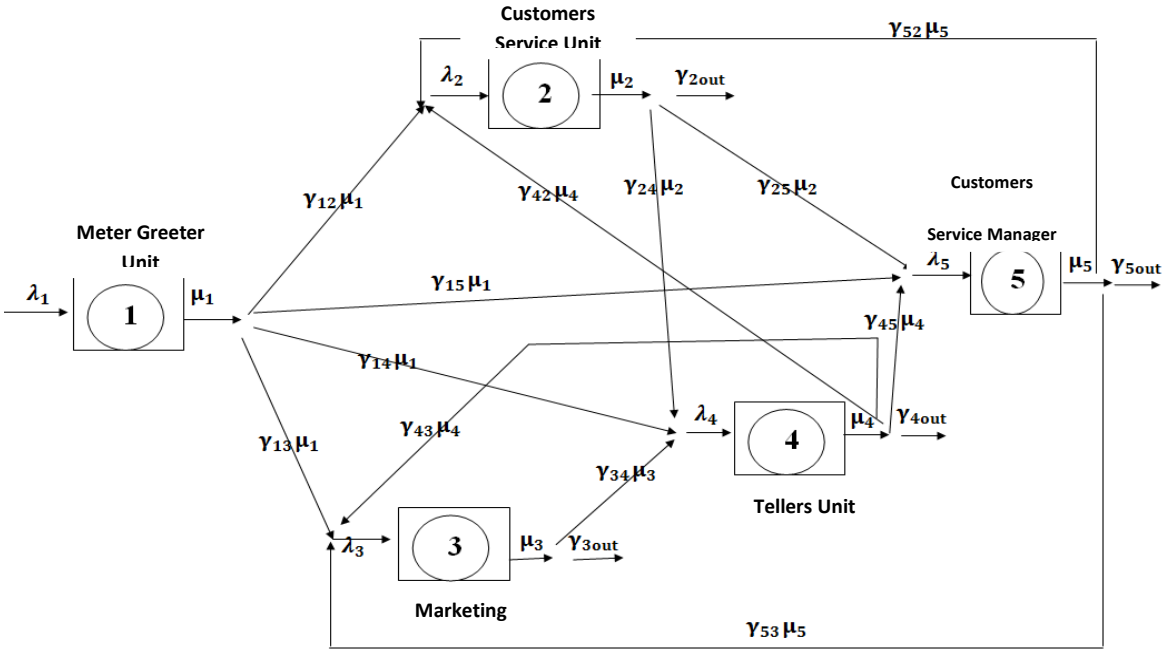



**Fig1. Schematic diagram of the First Bank, Plc. Bida network queuing system.**

Where: $\lambda_i$ is the arrival rate of the customer, for i = 1,2,….5

$\mu_i$ is the departure rate out of the system, for i = 1,2,… 5



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$\gamma_{ij}$ are the weight of moving from node i to node j.

$m_i$ is the number of servers at the various node in bank network system.

Node1, node2, node3, node4, node5 are the notation for the data presentation. They are denoted as follows: Meter Greeter unit denoted by node 1; Customer service unit denoted by node 2; Marketing unit denoted by node 3; Teller unit denoted by node 4; Customer Service Manager Unit denoted by node 5. The period of four (4) weeks together with number of arrivals and number of departures with an internal of 5 minutes.

**Where:** The arrival rate $\lambda_i = \frac{1}{mean\ number\ of\ arrival}$, for i = 1,2….5. (1)

The departure rate $\mu_i = \frac{1}{mean\ number\ of\ departure}$, for i= 1,2,…5 (2)

$$\rho = \frac{\lambda_i}{\mu_i},\ for\ i = 1,2, \dots \dots 6 \quad (3)$$

The expected number in the queue is given as: $l_q = \frac{\rho}{m-\rho}$, (4)

Where m stands for the number of servers at the node.

The expected waiting time in the queue is given as: $w_{qi} = \frac{l_q}{\lambda_i}$, (5)

The expected number of customer in the system is given as: $l_s = l_q + \rho$ (6)

Finally, the expected waiting time in the system for node 1-5 is given as:

$w_i = \frac{l_s}{\lambda_i}$, for i =1,2,…5 (7)

**Model Equations**

We obtain the model equation from the diagram in fig 3.1, where the banking system is model as an Open Jackson queuing network.

$$\lambda_2 = \gamma_{12}\mu_1 + \gamma_{42}\mu_4 + \gamma_{52}\mu_5 \quad (8)$$

$$\lambda_3 = \gamma_{13}\mu_1 + \gamma_{43}\mu_4 + \gamma_{53}\mu_5 \quad (9)$$



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$$\lambda_4 = \gamma_{14}\mu_1 + \gamma_{24}\mu_2 + \gamma_{34}\mu_3 \quad (10)$$

$$\lambda_5 = \gamma_{15}\mu_1 + \gamma_{25}\mu_2 + \gamma_{45}\mu_4 \quad (11)$$

Similarly;

$$\mu_1 = \gamma_{12}\mu_1 + \gamma_{13}\mu_1 + \gamma_{14}\mu_1 + \gamma_{15}\mu_1 \quad (12)$$

$$\mu_2 = \gamma_{24}\mu_2 + \gamma_{25}\mu_2 + \gamma_{2out}\mu_2$$

(13)

$$\mu_3 = \gamma_{34}\mu_3 + \gamma_{3out}\mu_3$$

(14)

$$\mu_4 = \gamma_{42}\mu_4 + \gamma_{43}\mu_4 + \gamma_{45}\mu_4 + \gamma_{4out}\mu_4 \quad (15)$$

$$\mu_5 = \gamma_{52}\mu_5 + \gamma_{53}\mu_5 + \gamma_{5out}\mu_5 \quad (16)$$

Where: $\gamma_{12}, \gamma_{13}, \gamma_{14}, \gamma_{15}, \gamma_{24}, \gamma_{25}, \gamma_{2out}, \gamma_{34}, \gamma_{30ut}, \gamma_{42}, \gamma_{43}, \gamma_{45}, \gamma_{4out}, \gamma_{52}, \gamma_{53}, \gamma_{5out}$ are to be determined.

From model equation (1 - 16) can be represented in the matrix form as:

$$\begin{bmatrix} \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_5 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & \mu_5 & 0 \\ 0 & 0 & \mu_1 & 0 & \mu_2 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_1 & 0 & \mu_2 & 0 & 0 & 0 & 0 & 0 & \mu_4 & 0 & 0 & 0 & 0 \\ \mu_1 & \mu_1 & \mu_1 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_2 & \mu_2 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_4 & \mu_4 & \mu_4 & \mu_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_5 & \mu_5 & \mu_5 \end{bmatrix} \begin{bmatrix} \gamma_{12} \\ \gamma_{13} \\ \gamma_{14} \\ \gamma_{15} \\ \gamma_{24} \\ \gamma_{25} \\ \gamma_{2out} \\ \gamma_{34} \\ \gamma_{3out} \\ \gamma_{42} \\ \gamma_{43} \\ \gamma_{45} \\ \gamma_{4out} \\ \gamma_{52} \\ \gamma_{53} \\ \gamma_{5out} \end{bmatrix} = \begin{bmatrix} \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} \quad (17)$$

**Mathematical Formulation for new Departure Rate:**

Since, the main aim of this research is to minimize the waiting time, there is need to introduce the new departure rate of each node.

Therefore, Model equation (8-11) will then generate equations below;

$$\lambda_2 = \gamma_{12}\mu_1 + \gamma_{42}\mu_2 + 0\mu_3 + 0\mu_4 + \gamma_{52}\mu_5 \quad (18)$$

$$\lambda_3 = \gamma_{12}\mu_1 + 0\mu_2 + 0\mu_3 + \gamma_{43}\mu_4 + \gamma_{52}\mu_5 \quad (19)$$

$$\lambda_4 = \gamma_{14}\mu_1 + \gamma_{24}\mu_2 + \gamma_{34}\mu_3 + 0\mu_4 + 0\mu_5 \quad (20)$$

$$\lambda_5 = \gamma_{15}\mu_1 + \gamma_{25}\mu_2 + 0\mu_3 + \gamma_{45}\mu_4 + 0\mu_5 \quad (21)$$



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$$\begin{bmatrix} \gamma_{12} & \gamma_{42} & 0 & 0 & \gamma_{52} \\ \gamma_{13} & 0 & 0 & \gamma_{43} & \gamma_{53} \\ \gamma_{14} & \gamma_{24} & \gamma_{34} & 0 & 0 \\ \gamma_{15} & \gamma_{25} & 0 & \gamma_{45} & 0 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{bmatrix}$$ Model equation (18-21) can be transform to matrix as in equation (22)

(22)

**The Mean arrival and Mean Departure Rate Analysis**

The mean arrival and service (departure) rates computed from the collected data are presented in Table 1.The results show that customer inflow varies across the service nodes, indicating uneven demand within the system. In particularly, Nodes 1, 3 and 4 recorded relatively high arrival rates compared to other nodes, suggesting heavier traffic at these service points.

**Table1.**The Mean arrival and mean departure time obtained from raw data Collected at the bank.

| | Node 1 | Node 2 | Node 3 | Node 4 | Node 5 |
|---|---|---|---|---|---|
| **Mean arrival** | 1.843 | 1 | 1.783 | 1.716 | 1.618 |
| **Mean departure** | 1.810 | 1 | 1.261 | 1.483 | 1.546 |

To find Solution for equation (17), we use the mean arrival and mean departure Time obtained from the raw data collected as presented in Table 1 above to Calculate the arrival rate and departure rare using equation (1) and (2) for $\lambda_i, i = 2,...,5$ *and* $\mu_i = i = 2,...,5$ then Substitute them in equation (17), hence we obtained the solution for equation (17) as presented in equation (23)

$$\begin{bmatrix} \boldsymbol{\gamma_{12}} \\ \boldsymbol{\gamma_{13}} \\ \boldsymbol{\gamma_{14}} \\ \boldsymbol{\gamma_{15}} \\ \boldsymbol{\gamma_{24}} \\ \boldsymbol{\gamma_{25}} \\ \boldsymbol{\gamma_{2out}} \\ \boldsymbol{\gamma_{34}} \\ \boldsymbol{\gamma_{3out}} \\ \boldsymbol{\gamma_{42}} \\ \boldsymbol{\gamma_{43}} \\ \boldsymbol{\gamma_{45}} \\ \boldsymbol{\gamma_{4out}} \\ \boldsymbol{\gamma_{52}} \\ \boldsymbol{\gamma_{53}} \\ \boldsymbol{\gamma_{5out}} \end{bmatrix} = \begin{bmatrix} 0.6543826458 \\ 0.1689061924 \\ 0.1070758869 \\ 0.06963527485 \\ 0.2420403511 \\ 0.5751615938 \\ 0.1827980551 \\ 0.6088290414 \\ 0.3911709586 \\ 0.3233371974 \\ 0.1167817581 \\ 0.0467082515 \\ 0.5131780648 \\ 0.2266694271 \\ 0.5466611462 \\ 0.2266694271 \end{bmatrix} \quad (23)$$

From equation (23), based on the result, the following deductions are made:



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At the point of node 1(Customer advisory unit), the weights $\gamma_{12}, \gamma_{13}, \gamma_{14}$ and $\gamma_{15}$ are $0.6543826458, 0.1689061924, 0.1070758869$ and $0.06963527485$ respectively shows that there is a high probability of a customer leaving node 1 (Customer advisory unit) to join the queue for service at node 2 (Customer service unit) than any other node. The least probability is that the customers leave node 1 to node 6.

At the point of node2 (Customer service unit), the weights $\gamma_{24}\ and\ \gamma_{2out}$ which are $0.2420403511$ and $0.1827980551$ which shows that there is a high probability that a customer leaves node 2 and goes directly to node 4 (Teller unit).

At the point of node3 (Marketing unit), the weights $\gamma_{34}\ and\ \gamma_{3out}$ which are $0.6088290414$ and $0.3911709586$ rrespectively shows that there is a high probability that a customer leaves node 3 (Marketing unit) to join the queue for service at node 4 (Teller unit). The least probability is that a customer leaves node 3 (Marketing unit) and moves out of the system.

At the point of node 4 (Teller's unit), the weights $\gamma_{42}, \gamma_{43}, \gamma_{45}, and\ \gamma_{4out}$ which are $0.3233371974, 0.1167817581, 0.0467082515$ and $0.5131780648$ respectively shows that of 2 weight, there is a higher probability that a customer leaves node 4 and go back to node 5 (i.e to see Customer service manager unit) than out of the system.

At the point of node5 (Customer service manager unit) the weights $\gamma_{52}, \gamma_{53}\ and\ \gamma_{5out}$ which are $0.2266694271$, $0.5466611462$ and $0.2266694271$ respectively shows that there is a high probability that the customers leave node 5 to join the queue for service either at node 2 or node 6.the least probability here is that a Customer leaves node 5 and goes out of the system.

At the point node6 (Account point), the weights $\alpha_{64}, \alpha_{65}\ and\ \alpha_{60ut}$ which are $0.38971315530$, $0.3847675569$ and $0.2255192878$ respectively shows that there is a high probability that the customers leave node 6 (Account unit) to join the queue for service either at node 4 or node 5. The least probability here is that a customer's leaves node 6 and goes out of the system.

From the information in above Table 1, we calculated the arrival and departure rates for each node, expected number of customers in the system, expected number of customers in the queue, expected waiting time in the system and expected waiting time in the queue.

For node 1, the arrival rate $\lambda_1 = \frac{1}{mean\ number\ of\ arrival} = \frac{1}{1.843} = 0.54$ .

**The departure rate for node is given as:** $\mu_1 = \frac{1}{mean\ number\ of\ departure} = \frac{1}{1.810} = 0.552$

Hence, we followed the same procedure to calculate for node 2- to node 5; the values obtained were presented in Table 2. Therefore Other values of the performance measure for all the nodes are computed and presented in Table 2.

N**ode 1:** $\rho = \frac{\lambda_1}{\mu_1} = \frac{0.54}{0.552} = 0.98$



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The expected number of customer in the queue is given as: $l_q = \frac{\rho}{m-\rho} = \frac{0.98}{2-0.98} = 0.96$

Where m stands for the number of servers at the node1 (Meter Greeter Unit).

The expected waiting time in the queue is given as $W_{q1} = \frac{l_q}{\lambda_1} = \frac{0.96}{0.54} = 1.77$ minute

The expected number of Customer in the system is given as: $l_s = l_q + \rho = 0.9 + 0.98 = 1.9$

The expected waiting time in the system for node 1 is given as: $W_1 = \frac{l_s}{\lambda_1} = \frac{1.9}{0.54} = 3.5$ minutes. These parameters calculated for node1 can also be done for nodes 2 – 5.

The expected waiting time in node 1 through node 5 were calculated as $w_1$ = 3.5 minutes for node 1, $w_2$ = 1.3 minutes for node 2, $w_3$ = 1.4 minutes, for node 3, $w_4$ = 12 minutes for node 4, $w_5$ =40.1 minutes for node 5.

Therefore, the total expected waiting time of the customer in the current system is computed as: $W_t = W_1 + W_2 + W_3 + W_4 + W_5 = 3.5 + 1.3 + 1.4 + 12 + 40.1 = 58.3 \cong 58\ Minutes$.

The values of these performance measures of the network system are presented in the Table 2 below:

**Table 2: Performance Measure for the Current Network System at the Bank**

| Number of Servers ($m_i$) | Probabilities ($\alpha_{ij}$) | $\rho_i$ | Lq | Ls | $W_q$ | $W_s$ |
|---|---|---|---|---|---|---|
| **2** | 0.6543826458.<br>0.1689061924.<br>0.1070758869.<br>0.0696352548 | 0.98 | 0.9 | 1.9 | 0.77 | 3.5 |
| **4** | 0.2420403511.<br>0.5751615938.<br>0.1827980551 | 1.00 | 0.3 | 1.3 | 0.33 | 1.3 |
| **2** | 0.6088290414.<br>0.3911709586 | 0.70 | 0.5 | 1.2 | 0.58 | 1.4 |
| **1** | 0.0.32333719.<br>0.0.116781758.<br>0.0.046708255<br>0.5131780648 | 0.86 | 6.1 | 6.9 | 11.20 | 12.0 |
| **1** | 0.2266694271.<br>0.5466611462.<br>0.2266694271 | 0.96 | 24.0 | 24.9 | 39.70 | 40.1 |
| **10** | | **4.50** | **31.8** | **36.2** | **52.58** | **58.3** |



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**Solution for New Departure Rate**

The new departure rate were obtained by adjusting the number of servers at each node such that the service capacity exceeds the arrival rate for each node, the minimum number of servers required was determined. We obtain .the new departure rate for the network system, from equation (22), this is done by substituting the values of arrival rate obtained in the previous section into equation (22) then we have equation (24)

$$\begin{bmatrix} 0.65 & 0.32 & 0 & 0 & 0.22 \\ 0.17 & 0 & 0 & 0.12 & 0.55 \\ 0.11 & 0.24 & 0.61 & 0 & 0 \\ 0.07 & 0.58 & 0 & 0.05 & 0 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} 1.00 \\ 0.56 \\ 0.58 \\ 0.62 \end{bmatrix} \quad (24)$$

After the necessary calculation, we obtained equations (25)

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} 8.04865 \\ 16.38245 \\ 16.26243 \\ 22.18840 \\ 7.056500 \end{bmatrix} \quad (25)$$

Therefore, to serve customer with the space interval of 5 minutes, we have to divided the values in equation (25) by 5. The recommended number of servers for each node is given in equation (26)

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} 1.6 \\ 3.3 \\ 3.3 \\ 4.4 \\ 1.4 \end{bmatrix} \cong \begin{bmatrix} 2 \\ 3 \\ 3 \\ 4 \\ 1 \end{bmatrix} \quad (26)$$

It indicates that The values in equation (26) have reduced the waiting time in the system to an acceptable level, node one will require a total of two servers, node two will require a total of three servers, node three will require a total of one server, node four will require a total of four servers, node five will require a total of one server. With the new estimated number of servers at each node given by equation (26). The arrival rates for each node are assumed to remain the same since we do not have control over it. A new departure rates and the expected waiting time for each node is estimated.

**The new departure rates for nodes are as follows:**

$\mu_1 = recommended\ departure\ rate\ per\ 5\ mins. = \frac{2}{5} = 0.4$

$\mu_2 = recommended\ departure\ rate\ per\ 5\ mins. = \frac{3}{5} = 0.6$



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$\mu_3 = recommended\ departure\ rate\ per\ 5\ mins. = \frac{1}{5} = 0.2$

$\mu_4 = recommended\ departure\ rate\ per\ 5\ mins. = \frac{4}{5} = 0.8$

$\mu_5 = recommended\ departure\ rate\ per\ 5\ mins. = \frac{1}{5} = 0.2$

**THE NEW EXPECTED WAITING TIME IN THE SYSTEM**

**Node 1:** $\rho = \frac{\lambda_1}{\mu_1} = \frac{0.54}{0.4} = 1.4$

The expected number of customers in the queue is given as: $l_q = \frac{\rho}{m-\rho} = \frac{1.4}{2-1.4} = 2.3$

Where m stands for the number of servers at the node1 (Meter greeter unit).

The expected waiting time in the queue is given as: $W_{q1} = \frac{l_q}{\lambda_1} = \frac{2.3}{0.54} = 4.26$ minute

The expected number of customers in the system is given as: $l_s = l_q + \rho = 2.3 + 1.4 = 3.7$

The expected waiting time of customers in the system for node 1 is given as: $W_1 = \frac{l_s}{\lambda_1} = \frac{3.7}{0.54} = 6.9$ minutes.

These parameters calculated for node 1 can also be done for node 2- 5.

The expected waiting time in node 1 through node 5 was calculated as $w_1$ = 6.9 minutes for node 1, $w_2$ = 3 minutes for node 2, $w_3$ = 30 minutes, for node 3, $w_4$ = 1.6 minutes for node 4, $w_5$ =7.7 minutes for node 5.

The total expected waiting time in the system after modification is

$$W_t = W_1 + W_2 + W_3 + W_4 + W_5 = 6.9 + 3 + 30 + 1.6 + 7.7 = 49.2 \cong 49\ Minutes$$

These values are presented in Table 3.

**Table 3: Showing all the results obtained after modification**

| Nodes i | Number of Servers ($m_i$) | $\rho_i$ | Lq | Ls | $W_q$ | $W_S$ |
|---|---|---|---|---|---|---|
| **1** | 2 | 1.4 | 2.3 | 3.7 | 4.26 | 6.9 |
| **2** | 3 | 1.7 | 1.3 | 3.0 | 1.30 | 3.0 |
| **3** | 3 | 2.8 | 14.0 | 16.8 | 25.00 | 30.0 |
| **4** | 4 | 0.7 | 0.2 | 0.9 | 0.30 | 1.6 |
| **5** | 1 | 0.8 | 4.0 | 4.8 | 6.45 | 7.7 |
| **Total** | **13** | **7.4** | **21.8** | **29.2** | **44.0** | **49.2** |

It can be observed from the table that the total number of serves is 13; this is optimal number of servers that can reduce waiting time to 49.2 minutes, approximately 50.minutes. These values are presented in Table below:



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**Table 4. Showing the comparison between current number of servers and optimal number of servers obtained**

| Nodes i | Current number of servers | Optimal number of servers obtained |
|---|---|---|
| **1** | 2 | 2 |
| **2** | 4 | 3 |
| **3** | 2 | 3 |
| **4** | 1 | 4 |
| **5** | 1 | 1 |
| **Total** | **10** | **13** |

**Results and Discussion**

The results show uneven customer distribution across service nodes, leading to congestion. The total waiting time in the existing system was 58 minutes, with Node 5 being the major bottleneck. After optimization, the waiting time reduced to 49 minutes due to improved server allocation. This confirms that inefficient server distribution was the main cause of delays.

The analysis showed that the existing system experiences significant delays, particularly at key service units such as the teller and customer service manager units, due to inadequate allocation of servers. By applying the queuing network model and adjusting the number of servers at each node, a more efficient system configuration was obtained. The results revealed that increasing the total number of servers from 10 to 13 leads to a noticeable reduction in the overall waiting time of customers.

The findings agree with previous studies (Gupta & Singh, 2022; Olanrewaju et al., 2023), which emphasize the importance of balancing service capacity with demand.

However, increasing servers may increase operational cost, suggesting a trade-off between efficiency and cost. The findings of this study highlight the importance of using analytical models, rather than relying on intuition or routine practice, in managing service systems. The Open Jackson model has proven to be an effective tool for identifying areas of congestion within the network.

**Conclusion**

This study examined the service system of First Bank Plc. Bida using the Open Jackson queuing network model in order to improve operational efficiency and reduce customer waiting time. This demonstrates that proper server allocation plays a critical role in improving service delivery.

In conclusion, optimizing the number of servers across different service units can significantly enhance system performance, reduce customer waiting time, and improve overall customer satisfaction.



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**Recommendation**

It is therefore recommended that bank management adopt following:

1. Banks should adopt data-driven queuing models.
2. Server allocation should be regularly reviewed.
3. Future studies should include cost optimization.

**REFERENCES:**

Adeleke, O. A., & Adebiyi, A. A. (2021). Application of queuing models in banking systems. *Journal of Operations Research*, 15(2), 45-60.

Emmanuel, B., & Lawal, A. (2020). Determination of optimal number of servers in banking systems. Federal University of Technology, Minna.

Gupta, R., & Singh, P. (2022). Optimization of service system using queuing theory. *International Journal of Applied Mathematics*, 34(1), 89-102.

Kembe, M., Onah, M., & Iorkegh, E. S. (2012). Waiting and service cost analysis of multi-server queuing systems. *International Journal of Scientific Research*, 1(8), 19-23.

Lawal, A., Adaji, I., & Abdulkadir, A. (2019). Waiting and service cost of multi-server systems. *Transactions of the Nigerian Association of Mathematical Physics*, 10, 177-184.

Olanrewaju, M. A., et al. (2023). Service efficiency in Nigerian banks using queuing models. *African Journal of Mathematics*, 18(3), 120-135.

Owoloko, A. E., et al. (2015). Application of Open Jackson queuing network. *Global Journal of Pure and Applied Mathematics,* 11, 2299 -2313.

Zhang, Y., et al. (2021). Queuing network analysis in service systems. Operations Research Letters, 49(4), 567-573.



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