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FORECAST ACCURACIES OF HYBRID OF BILINEAR AND EXPONENTIAL SMOOTH TRANSITION AUTOREGRESSIVE MODELS WITH GARCH MODELS

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Abstract

The study looks at the forecast accuracies of GARCH and Bilinear models on the one hand, and hybrids of Bilnear with GARCH (BL-GARCH) and ESTAR with GARCH (ESTAR-GARCH) models on the other hand. Necessary mathematical or theoretical frameworks were set up for the four models and were illustrated with inflation rate data from Nigeria (1994–2019). Stationarity tests conducted (Unit Root Test) show that at level (original data) the series is chaotic and volatile (non-stationary), but at [redacted] the first difference the series is stationary. This, therefore, paved the way for the analysis. In and out of sample, the forecast accuracies of all the models used were checked and compared. Information criteria such as AIC, BIC, and FPE were used, where it was seen that ESTAR-GARCH performs best. Also, the performance measure

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indices used revealed that ESTAR-GARCH was very outstanding. The same goes for variance analysis (ESTAR-GARCH had the least variance). The results show that the two hybrid models give exceptional performances; also, the GARCH model's performance is fairly good but not comparable to the hybrids. Exponential GARCH models perform better than Bilnear-GARCH models and GARCH models. However, the ESTAR-GARCH and BL-GARCH models are comparable and could be interchangeably used by would-be forecasters, investors, academia, and other would-be users.

INTRODUCTION

Engle (1982) developed the ARCH model, which was later extended to the GARCH model by Bollerslev (1986). The ARCH model expresses the conditional variance of a random variable as a linear function of its past squared realizations. Engle (1982) published a seminal paper, which was followed by other publications, such as Gouriéroux (1997) and Giraitis et al., which demonstrate the beauty and potential of the GARCH models in economic and financial fields (2007). GARCH models have already been shown to be capable of describing the chaotic nature of stock markets, exchange rate markets, and other financial markets in a number of studies over time. It is a well-known fact that either the Bilnear model or the traditional GARCH may fail to produce the correct findings as documented in the literature. These shortcomings prompted the researchers to look into the possibilities of mixing models (model hybridization) and comparing the resulting hybrid models to current models. Hybrid models outperform classical models, according to both empirical and theoretical findings. The ongoing study analyzes the effects of pairing bilinear with GARCH (BL-GARCH) and an exponential smooth transition autoregressive model (ESTAR-GARCH). Akintunde et al. (2013) combined bilinear and GARCH models, and the resulting novel model, designated BL-GARCH, outperformed each model individually. This claim is supported by indices of performance, model variance, and out-of-sample forecast performance. The authors determined that the novel model outperformed the classical GARCH model based on all of these criteria. Storti and Vitale (2003a)

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investigated the statistical features of the generic model and attempted to establish criteria for positive conditional variance and second-order stationarity. The autocorrelation function of the squared process for the BL-GARCH was developed using analytical formulae. Similarly, the unconditional fourth moment and the kurtosis coefficient, as well as requirements for the existence of the process's higher order moments, were provided. They used BL-GARCH to model continuously compounded returns and estimate the conditions for the volatility of financial time series data with other asymmetric GARCH models. Storti and Vitale (2003b) used an EM algorithm-based indirect maximum likelihood process. Despite the fact that the EM algorithm is a widely used computational method in statistics, it has some drawbacks, including low numerical convergence, because convergence to a maximum likelihood estimator is a function of careful starting point selection and the assumption of normality for the distributed data. The model, even with the auxiliary assumption of normally distributed errors, does not completely capture the very fat-tailed property of high frequency financial time series, as described by Baillie and Bollerslev (1991), McAleer (2005), and Zivot and Wang (2005).

MATHEMATICAL FRAME WORK

BILINEAR GARCH MODELS (BL-GARCH)

Let y_t be a mean-corrected series, then y_t follows a general GARCH (p,q) model if:

$y_t = \sigma_t \varepsilon_t$ and

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \quad (1)$$

Where again $\{\varepsilon_t\}$ is a sequence of iid random variables with mean 0 and variance 1.

The embodiment of bilinear process to equation (1) will capture the non-linear behaviour part of y_t, bilinear considers, the variability within explanatory variables as well as co-variations between the variable. So also, Bilinear-GARCH (BL-GARCH) models permit the capturing of asymmetries in the conditional

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variance of financial and economic-time series by means of synergy between past shocks and volatilities; thus, we postulate an augmented GARCH (p,q) model (BL-GARCH) as:

$$y_t = \sigma_t \varepsilon_t + \sum_{i=1}^{p}\sum_{j=i}^{q} \tau_{ij} y_{t-i}\varepsilon_{t-j} \tag{2}$$

The mean and variance of y_t for BL-GARCH as derived is

$$E(y_t) = \sigma_t E(\varepsilon_t) + \sum_{i=1}^{p}\sum_{j=i}^{q} \tau_{ij} E(y_{t-i}\varepsilon_{t-j})$$

$$= \begin{cases} 0 & \forall \mathrm{i} \neq \mathrm{j} \\ \sigma_{\varepsilon}^{2}\sum_{i=1}^{p}\tau_{ij} & ,\forall\ \mathrm{i}=\mathrm{j} \end{cases} \tag{3}$$

$$Var.(y_t) = \begin{cases} \dfrac{\alpha_0}{1-\sum_{\mathrm{l}=1}^{\mathrm{p}}(\alpha_{\mathrm{i}}+\beta_i)} & \forall\ \mathrm{i} \neq \mathrm{j} \\ \dfrac{\alpha_0}{1-\sum_{\mathrm{l}=1}^{\mathrm{p}}(\alpha_{\mathrm{i}}+\beta_i)} - \Delta^2(\theta) & \\ where\ \Delta^2(\theta) = \sigma_{\varepsilon}^{4}\left(\sum \tau_{ij}\right)^2 & \forall\ \mathrm{i}=\mathrm{j} \end{cases} \tag{4}$$

A statement of comparison needed to be made here to justify the need for the newly derived model and that of the conventional one. The reference here has to do with the variance of both models. Recall, the variance of classical GM is given as follows:

$$Var(y_t) = \begin{cases} \dfrac{\alpha_0}{1-\sum_{i=1}^{p}(\alpha_i+\beta_i)} & \forall\ \mathrm{i} \neq \mathrm{j} \end{cases} \tag{5}$$

So also, the variance of (BL-GARCH) as derived in (4) above is:

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$$Var(y_t) = \begin{cases} \dfrac{\alpha_0}{1-\sum_{i=1}^{p}(\alpha_i+\beta_i)} - \Delta^2(\theta), \forall \text{ i=j} \end{cases}$$

Where $\Delta^2(\theta) = \sigma_\varepsilon^4(\tau_{ij})^2$

The only condition where the classical and the newly derived model could be equated is when
the quantity $\sigma_\varepsilon^4 \tau_{ij} = 0$, so theoretically the use of BL-GARCH is justified, this will be justified empirically later.

ESTAR-GARCH

Exponential smooth transition autoregressive-GARCH (ESTAR-GARCH) MODELS
From the usual GARCH models we have

$$y_t = \sigma_t \varepsilon_t \qquad (6)$$

Where

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (7)$$

The EAR model may be generalized by permitting an intercept $\phi_0 \neq 0$ or $\theta_0 \neq 0$ or both. Another generalization is to abandon the condition of symmetry of the transition function around zero by adding a location parameter *c* and to allow the delay $d \geq 1$. The essence of the generalization is to make the EAR model location invariant. Thus, the function g_E becomes

$$g_E(\gamma, c; y_{t-d}) = 1 - \exp\left\{-\gamma(y_{t-d} - c)^2\right\}, \gamma \text{ f } 0 \qquad (8)$$

This model was termed the Exponential Smooth Transition Autoregressive (ESTAR) model by Terasvirta (1994), who also investigated parameter estimation procedures. It has the property that the transition function's

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minimum value is zero. This model has been used to model macroeconomic series such as the highly variable inflation series with success (Baharumshah and Liew, 2006).

Let $g_E = 1 - \exp\left\{-\gamma\left(y_{t-j} - c\right)^2\right\} \gamma > 0$

By virtue of equations (5) through (7), the smooth transition autoregressive GARCH model specification is

$$g_E = 1 - \exp\left\{-\gamma\left(y_{t-j} - c\right)^2\right\} = 1 - g_E = G_E$$

Such by taking the logarithm gives

$$G_E^{\bullet} = \log G_E = -\gamma\left(\sigma_t \varepsilon_t - c\right)^2$$
$$-\gamma\sigma_t^2\varepsilon_t^2 + 2\gamma c\sigma_t\varepsilon_t + \gamma c^2 \qquad (9)$$

The mean of $G_E^{\bullet}$

The expected value of $G_E^{\bullet}$ is

$$E\left[G_E^{\bullet}\right] = -\gamma E\left(\sigma_t^2\varepsilon_t^2\right) + \gamma c^2 \qquad = -\gamma E\left(y_t^2\right) + \gamma c^2$$

If y_t is mean deleted such that

$$E\left[G_E^{\bullet}\right] = -\gamma\sigma_y^2 + \gamma c^2;\ 0 \text{ otherwise}$$

$$E\left[G_E^{\bullet}\right] = -\gamma\sigma_y^2 + \gamma\mu_y + \gamma c^2$$

The variance of $G_E^{\bullet}$

Using similar arguments as used for BL-GARCH the variance of $G_E^{\bullet}$

$$\text{Var.}\left(G_E^{\bullet}\right) = \left\{\frac{\alpha_0}{1 - \sum\left(\alpha_i + \beta_j\right)} - \gamma\left(c^2 - \sigma_y^2\right)\right. \qquad (10)$$

PERFORMANCE MEASURE INDICES

The following criteria were used for the evaluation of the forecast performance of the series under consideration.

$$(i). \qquad RMSE = \sqrt{\left\{T^{-1}\sum_{t=1}^{T}\left(Y_t - \hat{Y}_t\right)\right\}}$$

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$$(ii). \quad MAE = T^{-1}\sum_{t=1}^{T}\left|Y_t - \hat{Y}_t\right|$$

$$(iii). \quad MAD = T^{-1}\sum_{t=1}^{T}\left(Y_t - \bar{Y}_t\right)$$

$$(iv). \quad MAPE = T^{-1}\sum_{t=1}^{T}\left|\frac{Y_t - \hat{Y}_t}{Y_t}\right| X\ 100$$

The smaller the values of those criteria listed above, the better the forecast performance; If the values obtained in the computations are close to zero, we say such a series is very good, suppose the results of the series are not consistent among the first three we used MAPE to serve as the benchmark.

ORDER DETERMINATION

Akaike Information Criterion (AIC) is used in the determination of the order (Akaike, 1974). Also, Final Prediction Error (F.P.E.). For the kth autoregressive model, the FPE criterion is given by

Final prediction Error is given as

$$FPE_{(k)} = \sigma_k^2[1 - k/N]$$

where σ_k^2 is the unbiased estimator of σ^2 using the k-th order model, that is

$$\sigma_k^k = RSS_k/(N-K)$$

Bayesian Information Criterion: is used for model selection. It is partly a product of the likelihood function and is equally related to Akaike information Criterion. BIC is expressed mathematically as

$$BIC_{(k)} = n\log\left(\hat{\sigma}_n^2\right) + k\ln(n)$$

Where $\hat{\sigma}_n^2$ is the error variance and defined as $\hat{\sigma}_n^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$

The minimum FPE or AIC is the best model for the data. AIC is known to perform better than FPE.

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DATA ANALYSIS AND INTERPRETATION

The inflation rate data was collected for Nigeria from 1994 to 2019. Software used in the analysis are E-view and set of programme to take care of ESTAR-GARCH and Bilnear-GARCH (BL-GARCH) which is not readily available in any software. The series in the study is a monthly data inflation rate data, of Nigeria. The essence of which is to determine the model that perform best among Classical GARCH and Bilinear models, on one part and Bilinear- GARCH and ESTAR- GARCH models on the other part for the period under study.

STATIONARITY

Stationarity of the series used was achieved using unit root methods. Augmented Dickey Fuller test was used to test for the presence of unit root. The series (inflation rates data) was examined at both level and at first difference. From the analysis, the results obtained show that at level the series was chaotic, violent and unstable. However, at first difference the series behaved and paved way for the analysis. as shown in the table 1 below:

TABLE 1: UNIT ROOT (AUGMENTED DICKEY-FULLER TEST)

Unit root	ADF-Test statistic	Critical value (5%)	Mackinnon prob
Level	0.6234	-2.8678	0.7543
First difference	-2.0137	-2.8678	0.0000

DESCRIPTIVE ANALYSIS

The nature of the data used for the study was examined using descriptive analysis procedure. The following data (as shown below) were obtained from the analysis.

TABLE 2: DESCRIPTIVE ANALYSIS

STATISTIC	VALUE
Mean	15.81083
Standard Error	1.988484
Median	11.56
Standard Deviation	15.40273

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Sample Variance	237.2441
Kurtosis	3.993936
Skewness	2.021856
Range	76.57
Minimum	-3.73
Maximum	72.84
Sum	948.65
Count	60
Confidence Level (95.0%)	3.978947

Table 2 above is the summary statistics of the series used for the study. The mean is positive while the standard error is low as observed from the table. Kurtosis shows the degree by which a distribution of random variables manifests or produces extreme values or outliers (extremely low or high). The series used for the study is leptokurtic as it contains excess positive kurtosis implying that the tail is fatter which the attribute of so many financial series is. The skewness observed for the series show that it is highly skewed indicating departure from symmetric properties; this is because most financial series exhibit asymmetric properties. The disparity between the minimum and maximum is very high indicating that the volatility of the series.

MODEL VARIANCES

From the analysis the following model variances were obtained. The least variances was produced by ESTAR-GARCH closely followed by BL-GARCH, while classical GARCH model produced the worst as shown here under:

Table 3: MODEL VARIANCES

MODEL	GARCH	BILINEAR	BL-GARCH	ESTAR-GARCH
VARIANCES	27.5621	24.7612	9.2341	6.3217 *

MODEL ADEQUACY

To examine the adequacy of the series under study, the following information criterion (Akaike information criteria, Bayesian information criteria and Final

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prediction error) were used. From the analysis the following were obtained and tabulated as shown below

TABLE 4: MODEL ADEQUACY

INFORMATION CRITERIA MODELS	GARCH	BILINEAR	BL-GARCH	ESTAR-GARCH
AIC	8.4132	0.4732 *	5.2311	4.0231
BIC	7.2389	1.8171	5.3214	5.9318 *
FPE	9.2311	2.4329	5.2719	5.3176 *

From the above table ESTAR-GARCH was best in BIC and FPE while Bilinear model was best when AIC was considered. The conclusion drawn here is that ESTAR-GARCH is best if the criterion of information is used, this is followed by Bilinear model.

MODELS VERSUS PERFORMANCE MEASURE INDICES

The study looks at both in and out of sample forecast performance measures indices using RMSE, MAE, MAPE and MAD. For the two situations (in and out of sample forecast performance) as shown here under. Table 3.51 reveals that of the four models used, ESTAR model outperformed other models; this is followed by BL-GARCH, the same results was obtained for out-of- sample forecast performance, where ESTAR-GARCH performed far better than other models as shown in table 3.5.2

IN-SAMPLE FORECAST ANALYSIS

TABLE 5: IN-SAMPLE FORECAST ANALYSIS

FORECAST INDICES MODELS	GARCH	BILINEAR	BL-GARCH	ESTAR-GARCH
RMSE	0.0421	0.0401	0.0231	0.0092 *
MAE	0.0092	0.0081	0.0034 *	0.0072
MAPE	122.6266	109.2341	110.7821	99.7321 *

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MAD	0.9432	0.9562	0.9653	0.9832 *

OUT OF SAMPLE FORECAST ANALYSIS

TABLE 6: OUT OF SAMPLE FORECAST ANALYSIS

FORECAST INDICES MODELS	GARCH	BILINEAR	BL-GARCH	ESTAR-GARCH
RMSE	0.0472	0.0412	0.0392	0.0032 *
MAE	0.0096	0.0091	0.0061 *	0.0083
MAPE	145.3214	124.3216	122.4118	100.0192 *
MAD	0.9821	0.9862	0.9901	0.9952 *

SUMMARY OF RESULTS/CONCLUSION

Tables 2 and 3 depict all properties contained in the series and variance under study. Such information includes kurtosis, skewness variance, and others. The series was leptokurtic and highly skewed, indicating a departure from symmetric properties. The variance analysis shows that ESTAR-GARCH produces the least variance, while the highest was produced by the GARCH model. Table 4 is a table of information criteria where AIC, BIC, and FPE were used. Analysis revealed that Bilinear had the best result using AIC as a criterion while ESTAR-GARCH had the least in BIC and FPE. Tables 5 and 6 show both in- and out-of-sample forecast evaluation. From the table, it was shown that ESTAR-GARCH was best

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