



SIMULATING THE EFFECTS OF EXPONENTIAL INCREASE IN BLOOD MASS FLOW AND METABOLISM DURING PHYSICAL ACTIVITY

**SAIDU YAKUBU VULEGBO¹, MUSA
BAWA², ALIYU YAHAYA BADEGGI²,
ABDUL MOHAMMED³ &
ABUBAKAR ABDULKADIR²**

¹Department of Mathematics Federal Polytechnic Bida, Niger State. ²Department of Mathematical Sciences Ibrahim Babangida University Lapai, Niger State. ³Department of Human Kinetics & Health Education Ibrahim Babangida University Lapai, Niger State

Corresponding Author:

saidu.yakubu@fedpolybida.edu.ng

DOI: <https://doi.org/10.70382/mejnsar.v7i9.026>

Abstract

Physical exercise is simply an activity that requires physical effort, carried out to sustain or improve health, fitness and wellbeing. This study is mainly aimed at simulating the effects of exponential increase

in blood mass flow and metabolism during physical activity. The existing governing equation on physical exercise was modified in other to formed model equation that governs the phenomena under consideration and Olayiwola's Generalized Polynomial

Keywords;

Physical exercise,
Blood mass flow,
Metabolism,
Modeling &
OGPAM

Approximation Method (OGPAM) was used to solve the proposed model. It was observed that during physical exercise, an increase in metabolism and blood mass flow rate improves the tissue

temperature profile $\theta(x, t)$ along the specified distance x and against time t . The results also demonstrate that the basal metabolic rate and tissue temperature profile both drops with time as the body carries out life-sustaining function.

INTRODUCTION

Physical exercise in recent times is simply seen to be any form of activity that requires physical effort, carried out to sustain or improve health, fitness and wellbeing. Exercise improves mood, regulates weight, helps you avoid gaining too much weight or maintain the weight you've dropped, improves sleep quality, fights sickness and health issues, increases energy, and rekindles your sexual desire. During physical exercise, the body needs more energy when engaging in different sorts of physical exercise, which increases the metabolic rate and the pace at which heat is produced. An increase in activity level during physical exercise increases the blood mass flow rate which later becomes so aberrant. As this trend continues it triggers sweat evaporation which comes out of the skin in form of water and electrolyte.

Gurung and Shrestha (2017) examined that the heat being generated while engaging in physical exercise can be lost and gain back via conduction, convection, blood perfusion and it is completely loose out via evaporation.

The product of a volumetric perfusion rate and the temperature differential between local tissue and arterial blood determines the rate of heat transfer between blood and tissue. The body needs more food when a person engages in different types of physical activity, which increases the metabolic rate and the pace at which heat is produced. With a sweat output rate of up to 1-2 liters per hour, those who exercise in hotter climes are more likely to lose fluid and electrolytes and get dehydrated (Wiarto, 2013). Electrolyte balance is crucial in human body simply because it impacts fluid balance and cell function and two most significant cations are potassium and sodium.

Both have a direct impact on cell activity and alter the osmotic pressure of the intracellular and extracellular fluid. Muscle contraction can be influenced by the body's minerals and electrolytes. Muscle cells need the energy generated by mitochondria to contract (Barthwal, 2004).

LITERATURE REVIEW

In recent times, a lot of efforts have been made to model different forms of behaviors of human body during physical exercise choosing parameters of interest and many more researches currently ongoing in other fields of mathematics.

Saidu et al. (2024) examined the effect of temperature-dependent thermal conductivity in the human body during physical and the results obtained showed that when the body performs life-sustaining functions, the basal metabolic rate decreases against time and also decreases the tissue temperature profile $\theta(x, t)$. The amount of perspiration produced by the body during physical activity allows for heat released by evaporating sweat from the skin's surface, which cools the body. Therefore, body temperature regulation during exercise is crucial for maintaining homeostasis and preventing heat related illnesses. This is because the metabolic rate increases logarithmically and the blood mass flow rate increases periodically.

In the same vein, Shrestha (2020) stated that the temperature differential between the skin and the surrounding environment determines the total amount of heat loss from the human body. Normal blood flow occurs when the body is at rest, however during physical activity, this flow becomes extremely abnormal. Garcia (2022a) asserts that extracurricular activities are crucial to the learning process.

Additionally, studies have demonstrated that youth physical activity levels are significantly impacted by well-designed school physical education programs (Jacob et al., 2021). In a similar vein, Chen (2023) wrote on individual variances in temperature regulation during exercise. It was discovered that intense

exercise causes the body to produce a lot of heat energy, which raises body temperature considerably. The core temperature of the body can therefore rise by a few degrees due to vigorous activity, which can produce over 1000 watts of heat. A key factor in the body's heat dissipation process is sweat rate.

Sweat evaporation off the skin's surface helps the body to release heat, which cools the body. This is made possible by the amount of sweat the body produces during physical activity. Therefore, preserving homeostasis and avoiding heat-related disorders during exercise depend heavily on body temperature regulation. Sweating rates, however, can differ greatly from person to person for a number of reasons, such as genetics, body composition, level of fitness, and hydration. For example, research indicates that men often perspire more than women, which could account for women's increased vulnerability to heat-related illnesses while exercising (Gagnon et al., 2017).

Banuelos et al. (2021) modeled the long term effect of thermoregulation on human sleep and discovered that increases in activity level can increase fatigue which in turns enhances sleep

World health organization physical activity guide lines for sedentary behaviors (WHO, 2020) recommends that we:

- (i) Refrain from exercising in extreme heat, particularly when there is a lot of humidity.
- (ii) Drink water before, during, and after physical activity to stay hydrated and.
- (iii) Steer clear of physical contact-based activities, activities that increase the danger of falling, or activities that could restrict oxygenation (e.g., activities at high altitude, while not regularly living at high altitude). In order to prevent and control non-communicable diseases (NCDs) like cardiovascular disease, type 2 diabetes, and some types of cancer, regular physical activity is essential. In addition to preventing cognitive decline and the symptoms of

anxiety and depression, physical activity also helps people maintain a healthy weight and improve their general well-being. Global estimates indicate that 27.5% of adult and 81% of adolescents do not meet the 2010 WHO recommendations for physical activity with almost no improvements seen during the past decades. Guthold *et al.* (2020) wrote that in most countries, girls and women are less active than boys and men, and that there are significant differences in levels of physical activity between higher and lower economic groups, and between countries and regions.

MATERIAL AND METHOD

MODEL MODIFICATION

In accordance with the findings of Saidu *et al.* (2024), which describe how temperature-dependent thermal conductivity affects physical activity: The use of mathematical modeling

Thus,

$$\frac{\partial \theta}{\partial t} = \varepsilon \frac{\partial}{\partial x} \left((1 + \sigma \theta) \frac{\partial \theta}{\partial x} \right) + m(t)(\gamma - \theta) + s(t) \quad (1)$$

With initial and boundary conditions as

$$\theta(x, 0) = 1, \quad \frac{\partial \theta}{\partial x} \Big|_{x=0} - \gamma_1 \theta = \gamma_2 LE, \quad \theta \Big|_{x=1} = 1 \quad (2)$$

Where,

$$\varepsilon = \frac{k_0 t_0}{\rho c b^2}, \quad s(t) = \frac{s(t) t_0}{\rho c (T_b - T_a)}, \quad \gamma = \frac{(T_A - T_a)}{(T_b - T_a)}, \quad \gamma_1 = \frac{bh}{k^*}, \quad \gamma_2 = \frac{h}{k^* (T_b - T_a)}$$

and

θ =dimensionless temperature, t = time spent during physical exercise, ε = diffusion term, σ = thermal conductivity, $m(t)$ = blood mass flow rate, γ = radius of the skin at the dermal $s(t)$ = rate of metabolism, γ_1 = radius of the skin at the epidermal , γ_2 = radius of the skin at the SST, L = Latent heat of evaporation and E = rate of sweat evaporation in tissue.

According to Saidu et al. (2017) the exponential increases in blood mass flow

rate is given by:

$$m(t) = (m_0 + m_0 e^{-\alpha t}) \quad (3)$$

Where,

m_0 = initial value of the blood mass flow rate before the exercise commences,
 α = Metabolic control parameter when exercise is ongoing.

Also, in the work of Gurung & Shrestha (2017) the exponential increases in

metabolic rate is given by:

$$s(t) = BMR + \beta(1 - e^{-\alpha t}) \quad (4)$$

Where,

BMR = basal metabolic rate, β = exercise control parameter

Incorporating equation (3) and (4) into equation (1) gives

$$\frac{\partial \theta}{\partial t} = \varepsilon \frac{\partial}{\partial x} \left((1 + \sigma \theta) \frac{\partial \theta}{\partial x} \right) + (\gamma - \theta)(m_0 + m_0 e^{-\alpha t}) + (BMR + \beta(1 - e^{-\alpha t}))$$
$$\theta(x, 0) = 1, \quad \frac{\partial \theta}{\partial x} \Big|_{x=0} - \gamma_1 \theta = \gamma_2 LE, \quad \theta \Big|_{x=1} = 1. \quad (5)$$

The above equation (5) is the proposed model describing the effect of exponential increase in blood mass flow rate and metabolic rate during physical exercise.

METHODOLOGY

The Olayiwola Generalized Polynomial Approximation Method (OGPAM, 2022) is the proposed method of solution. One of the most straightforward

and in some situations, precise techniques for solving parabolic equations in cases of considering slabs, cylinder and spherical geometry is the generalized polynomial approximation method developed by Olayiwola. The definition of the parabolic equations is given by

$$\frac{\partial \phi}{\partial t} = \frac{k}{r^n} \frac{\partial}{\partial r} \left(r^n \frac{\partial \phi}{\partial r} \right) + F(r, t, \phi), \quad t > 0, \quad r \in \Omega \quad (\Omega \subset R^1, R^2 \text{ or } R^3) \quad (6)$$

with the initial condition,

$$\phi(r, 0) = f(r) \quad (7)$$

And the boundary conditions,

$$\alpha_1 \frac{\partial \phi}{\partial r} \Big|_{r=a} + \beta_1 \phi \Big|_{r=a} = g_1(t), \quad \alpha_2 \frac{\partial \phi}{\partial r} \Big|_{r=b} + \beta_2 \phi \Big|_{r=b} = g_2(t) \quad (8)$$

where $F(r, t, \phi)$ are the other terms in equation (5), $n = 0$ for a slab, $n = 1$ for cylinder, $n = 2$ for sphere, $f(r)$, $g_1(t)$, $g_2(t)$ are three known functions, $n, k, \alpha_1, \alpha_2, \beta_1, \beta_2$ are constants, and $a \leq r \leq b$ is the boundary of Ω . We then compared the partial differential equation of the form equations (6)–(8) with equation (5) together with its beginning and boundary conditions in order to find an analytical solution of the suggested model, thereby yielding an approximate solution:

Thus,

Let $0 \ll \sigma \ll 1$ such that

$$\theta(x, t) = \theta_0(x, t) + \sigma \theta_1(x, t) + \dots \quad (9)$$

Substituting equation (9) into equation (5) gives.

σ^0 :

$$\sigma^1 : \frac{\partial \theta_0}{\partial t} = \varepsilon \frac{\partial^2 \theta_0}{\partial x^2} + (\gamma - \theta_0)(m_0 + m_0 e^{-at}) + (BMR + \beta(1 - e^{-at}))$$

$$\theta_0(x, 0) = 1, \quad \frac{\partial \theta_0}{\partial x} \Big|_{x=0} - \gamma_1 \theta_0 \Big|_{x=0} = \gamma_2 LE, \quad \theta_0 \Big|_{x=1} = 1. \quad (10)$$

$$\frac{\partial \theta_1}{\partial t} = \varepsilon \frac{\partial^2 \theta_1}{\partial x^2} + \varepsilon \theta_0 \frac{\partial \theta_0}{\partial x} + \varepsilon \left(\frac{\partial \theta_0}{\partial x} \right)^2 - \theta_1 (m_0 + m_0 e^{-at})$$

$$\theta_1(x, 0) = 0, \quad \frac{\partial \theta_1}{\partial x} \Big|_{x=0} - \gamma_1 \theta_1 \Big|_{x=0} = 0, \quad \theta_1 \Big|_{x=1} = 0. \quad (11)$$

SOLUTION VIA OGPAM

Now consider equation (10) and compared with equation (6)-(8), we have

$$k = \varepsilon, n = 0, \phi = \theta_0, f(r) = 1, r = x, f(r, t, \phi) = (\gamma - \theta_0)(m_0 + m_0 e^{-at}) + (BMR + \beta(1 - e^{-at})),$$

$$\alpha_1 = 1, \beta_1 = -\gamma_1, g_1(t) = \gamma_1 LE, \alpha_2 = 0, \beta_2 = 1, g_2(t) = 1, a = 0, b = 1.$$

Thus, equation (10) has a solution of the form

$$\theta_0(x, t) = \gamma_2 LEx + (1 - \gamma_2 LE)x^2 + \theta_0 \Big|_{x=0} (1 + \gamma x - (1 + \gamma_1)x^2) \quad (12)$$

Where,

$$\theta_0 \Big|_{x=0} = c_0 + c_1 t + c_2 t^2 + (c_3 + c_4 t^2) e^{B_0 t} \quad (13)$$

And,

$$\begin{aligned}
 B_0 &= \frac{1}{A} \left(\varepsilon \gamma_1 - \alpha m_0 \left(\frac{2}{3} + \frac{1}{6} \gamma_1 \right) \right), B_1 = \frac{1}{A} \alpha m_0 \left(\frac{2}{3} + \frac{1}{6} \gamma_1 \right), \\
 B_2 &= \frac{1}{A} \left(2\varepsilon - \varepsilon \gamma_2 LE + \left(\alpha m_0 \left(\gamma - \frac{1}{3} - \frac{1}{6} \gamma_2 LE \right) + BMR \right) \right), \\
 B_3 &= \frac{1}{A} \alpha \left(\beta - m_0 \left(\gamma - \frac{1}{3} - \frac{1}{6} \gamma_2 LE \right) \right), C_0 = \frac{B_1 B_2}{B_0^3} - \frac{B_2}{B_0} - \frac{B_3}{B_0^2} \\
 C_2 &= \left(\frac{B_1 B_2}{B_0^2} - \frac{B_3}{B_0} \right) t + \frac{B_1 B_2}{B_0^2} t^2, C_3 = \left(1 + \frac{B_2}{B_0} + \frac{B_3}{B_0^2} - \frac{B_1 B_2}{B_0^3} \right) \\
 &\left(\beta - m_0 \left(\gamma - \frac{1}{3} - \frac{1}{6} \gamma_2 LE \right) \right), c_4 = \frac{B_1}{2} c_3.
 \end{aligned} \tag{14}$$

Also, comparing equation (11) with equation (6) - (8) we have,

$$\begin{aligned}
 k &= \varepsilon, n = 0, \phi = \theta_0, f(r) = 0, r = x, f(r, t, \phi) = (m_0 + m_0 e^{-\alpha t}) \theta_1 + c_5 + c_6 \theta_0 \Big|_{x=0} + c_7 (\theta_0 \Big|_{x=0})^2, \\
 \alpha_1 &= 1, \beta_1 = -\gamma_1, g_1(t) = 0, \alpha_2 = 0, \beta_2 = 1, g_2(t) = 0, a = 0, b = 1.
 \end{aligned}$$

Then, equation (11) has a solution of the form

$$\theta_1(x, t) = (1 + \gamma_1 x - (1 + \gamma_1) x^2) \theta_1 \Big|_{x=0} \tag{14}$$

Where,

$$\begin{aligned}
 \theta_1 \Big|_{x=0} &= (C_6 + C_{11} t + C_{21} t^2) e^{B_0 t} + (C_{20} + C_{22} t + C_{23} t^2) e^{2B_0 t} - \\
 &(C_{17} + C_{18} t + C_{19} t^2)
 \end{aligned}$$

And,

$$c_6 = \varepsilon (2(\gamma_1\gamma_2LE + (1-\gamma_2LE))) + 6(\gamma_1(1-\gamma_2LE) - \gamma_2LE(1+\gamma_1)) -$$

$$12(1+\gamma_1)(1-\gamma_2LE), C_{11} = \frac{1}{A}(C_3C_6 + 2C_0C_3C_5), C_{17} = \left(\frac{C_8}{B_0} + \frac{C_9}{B_0^2} + \frac{2C_{10}}{B_0^3}\right),$$

$$C_{18} = \left(\frac{C_9}{B_0} + \frac{2C_{10}}{B_0^3}\right), C_{19} = \left(\frac{C_{10}}{B_0} + \frac{1}{2}B_1C_{17}\right), C_{20} = \left(\frac{C_{12}}{B_0} + \frac{2C_{15}}{B_0^2}\right), C_{21} = \left(\frac{C_{13}}{2} + \frac{1}{2}B_1C_{16}\right),$$

$$C_{22} = \frac{2C_{15}}{B_0^2}, C_{23} = \left(\frac{C_{15}}{B_0} + \frac{1}{2}B_1B_1C_{20}\right)$$

Thus, solution to equation (5) is of the form

$$\theta(x,t) = \theta_0(x,t) + \sigma\theta_1(x,t) \tag{15}$$

RESULTS AND DISCUSSIONS

Table 1: Numerical data's used for Simulation of the result

S/NO	SYMBOL	MEANING	FIGURE	REFERENCE
1	m_0	Blood Mass Flow Rate	1	Saidu M.Tech Thesis 2017
2	E	Rate of Sweat Evaporation in Tissue	0.00048g/cm ² min	Shrestha et al., 2017
3	ε	Diffusion Term	0.2	Saidu etal. (2024)
4	α	Metabolic Control Parameter	0.5	Saidu etal. (2024)
5	s_0	Metabolic Rate	0.2	Saidu etal. (2024)
6	σ	Thermal Conductivity	0.4	Saidu M.Tech Thesis 2017
7	β	Exercise Control Parameter	2	Saidu M.Tech Thesis 2017

8	L	Latent Heat	579	Gokul et al. (2015)
9	BMR	Basal Metabolic Rate	$0.3165 \text{ cal/cm}^2 \text{ min}$	Brashaw et al. (2006)
10	γ	Radius of The Skin (Dermis)	1	Saidu et al. (2024)
11	γ_1	Radius of The Skin (Epidermis)	0.5	Saidu et al. (2024)
12	γ_2	Radius of The Skin (Subcutaneous Tissue)	0.3	Saidu et al. (2024)

Sources: As referenced

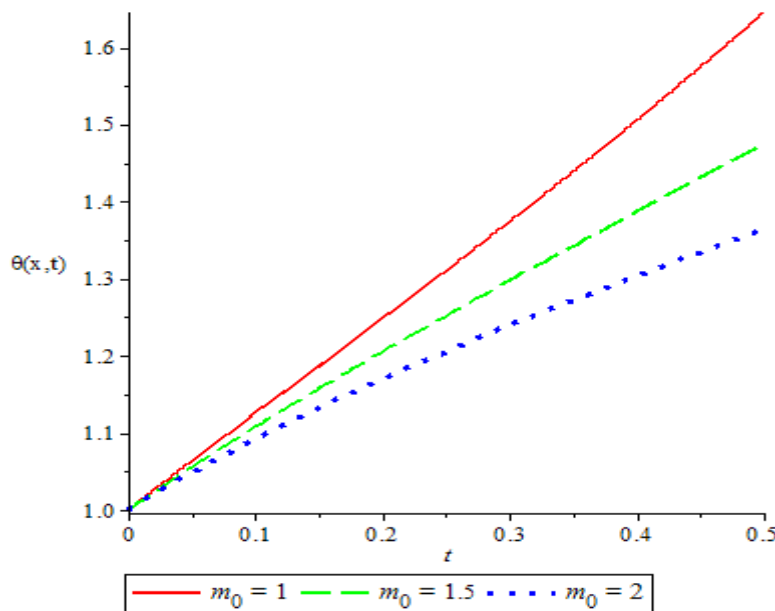


Figure 1: This graph illustrates how physical exercise increases blood mass flow rate, which in turn improves the body tissue temperature profile $\theta(x,t)$ at time t .

Figure 1: Variation of blood mass flow rate m_0 in human body during physical exercise at time t

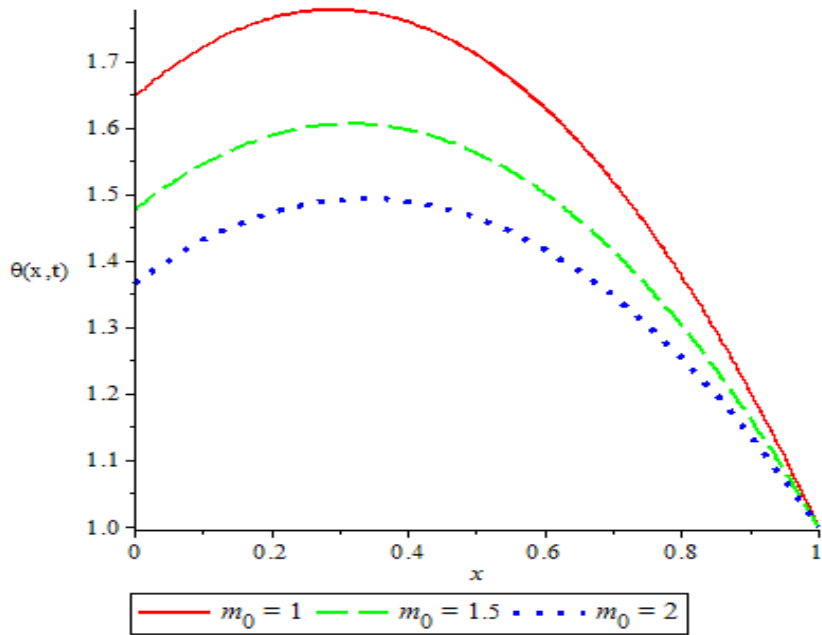


Figure 2: This graph illustrates how physical exercise raises blood mass flow rate, which improves the human tissue temperature profile $\theta(x,t)$ across a large distance x .

Figure 2: Variation of blood mass flow rate m_0 in human body during physical exercise along the given distant x

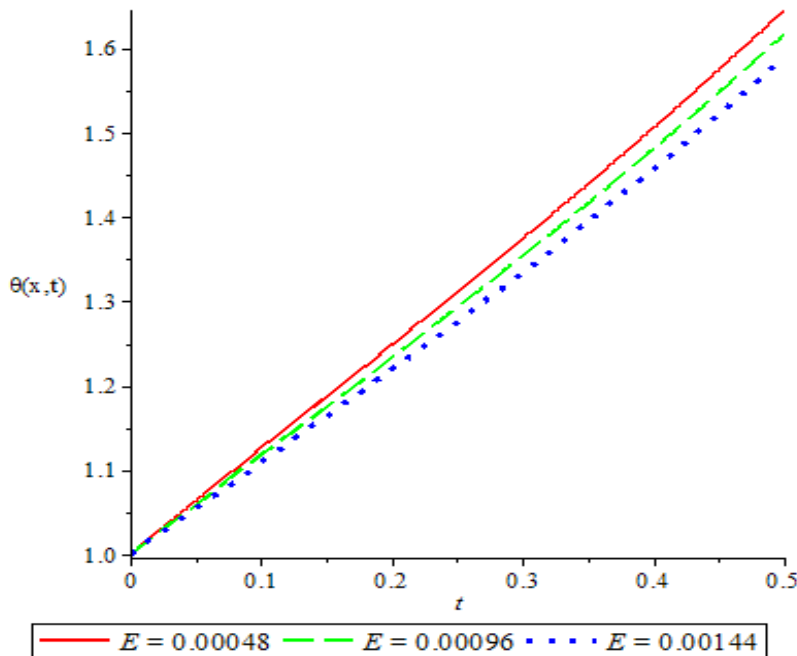


Figure 3: The graph here shows that there is an increase in the rate of sweat evaporation E during physical exercise. This increase in the rate of sweat evaporation enhances the body tissue temperature profile $\theta(x,t)$ against time t .

Figure 3: Variation of Sweat evaporation in human body E during physical exercise at time t

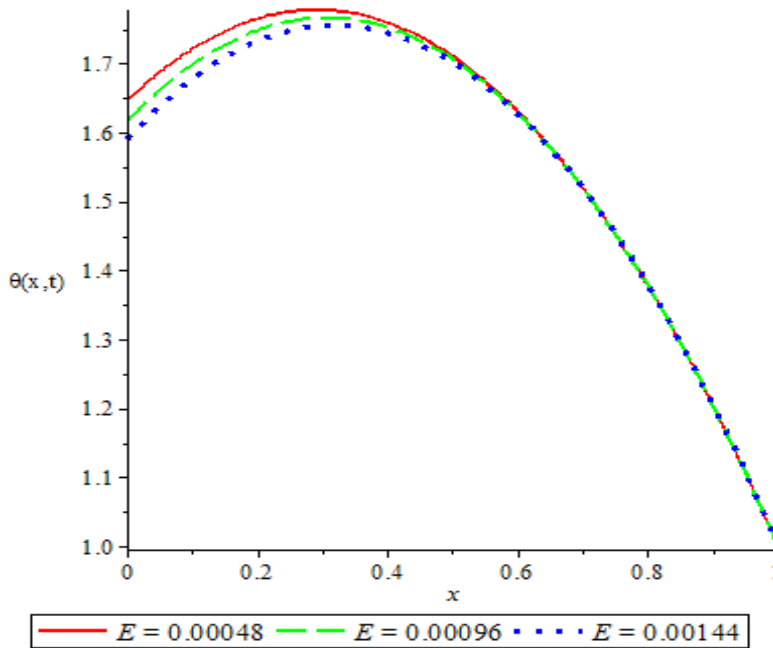


Figure 4: The graph here shows that there is increase in the rate of sweat evaporation E during physical exercise and it as well enhances the body tissue temperature profile $\theta(x, t)$ a long distance x .

Figure 4: Variation of Sweat epevoration in human body E during physical exercise along the given distance x .

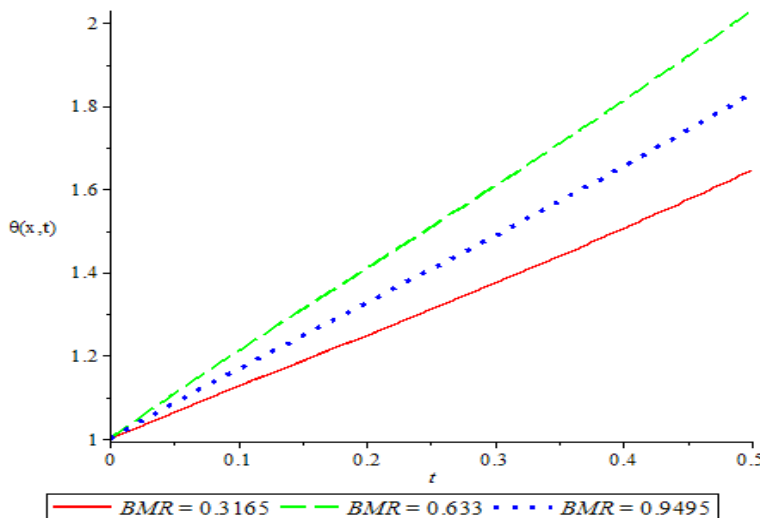


Figure 5: This graph illustrates how physical exercise lowers both the human tissue temperature profile $\theta(x, t)$ at time t and the basal metabolic rate (BMR) as the body performs life sustaining function.

Figure 5 : Variation of basal metabolic rate BMR in human body during physical exercise at time t

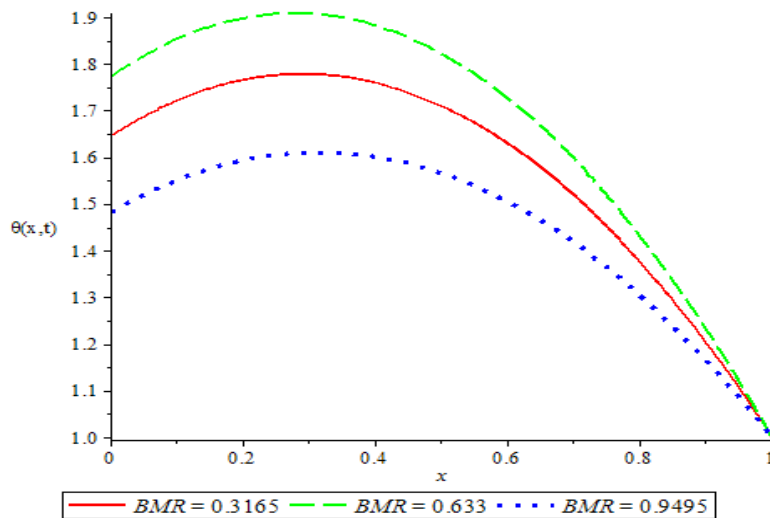


Figure 6: This graph illustrates how exercise increases the human tissue temperature profile $\theta(x,t)$ over distance x and causes a modest drop in basal metabolic rate (BMR).

Figure 6: Variation of basal metabolic rate in human body BMR during physical exercise along distance x

CONCLUSION

The physiology role played by the rate of sweat evaporation, basal metabolic rate and blood mass flow rate, during the physical exercise has been described. The findings indicate that a higher degree of activity raises both the tissue temperature profile and the blood mass flow rate. Similarly, we have seen that a higher degree of activity raises the pace at which perspiration evaporates from tissue and improves the temperature profile of that tissue. The information acquired will be crucial for students studying human kinetics, health educators, military and paramilitary units, and other relevant fields.

RECOMMENDATION

Human kinetic experts, military and paramilitary officials can utilize the information from this research report as guidance when training and during working out sessions.

REFERENCES

- Banuelos, S., Bestb, J., Huguetc, G., Prieto-Langaricad, A., Pamela B. P., & Wilson B. (2021). Modeling the Long Term Effects of Thermoregulation¹ on Human Sleep.
- Barthwal, M. S. (2004). Analysis of arterial blood gases a comprehensive approach. Review article. JAPI; 52573–577.
- Chen, D. (2023): Temperature regulation during exercise and the individual differences <https://dio.org/10.105/shsconf/20217403013>

- Gagnon, D., Kenny, G. P., & Havenith, G. (2017). Sex differences in thermoregulation during exercise. *4(3)*, 292-316.
- Garcia, M. B. (2022a). Hackathons as Extracurricular Activities: Unraveling the Motivational Orientation Behind Student Participation. *Computer Applications in Engineering Education*, *30(6)*, 1903-1918. <https://doi.org/10.1002/cae.22564>
- Guthold, R., Stevens, G. A., Riley, L.M., & Bull, F.C. (2020). Global trends in insufficient physical activity among adolescents: a pooled analysis of 298 population-based surveys with 1.6 million participants. *Lancet Child Adolesc Health*. *4(1)*:23-35.
- Jacob, C. M., Hardy-Johnson, P. L., Inskip, H. M., Morris, T., Parsons, C. M., Barrett, M., Hanson, M., Woods-Townsend, K., & Baird, J. (2021). A Systematic Review and Meta-Analysis of School-Based interventions with Health Education to Reduce Body Mass Index in Adolescents Aged 10 to 19 Years. *International Journal of Behavioral Nutrition and Physical Activity*, *18(1)*, 1-22. <https://doi.org/10.1186/s12966-020-01065-9>.
- Olayiwola, R. O. (2022): Solving parabolic equations by Olayiwola's generalized polynomial approximation method, *International Journal of Mathematical Analysis and Modeling*, *5(3)*: 24 – 43.
- Saidu, Y. V., Musa, B., Aliyu, Y. B., & Abdul, M. (2024). The effect of temperature dependents thermal conductivity during physical exercise: Mathematical modeling approach. *International Journal of Modeling & applied science research published by Cambridge research and publication* *3(9)*, 88-94.
- Saidu, Y. V., Olayiwola R. O., & Mahmood H., (2017). Simulation of the effect of physical exercise on temperature distribution in the peripheral regions of human limbs SPSBIC Conference Federal University of technology Minna.
- Shrestha, D. C., Acharya, S., & Gurung, D. B. (2020). Modeling on Metabolic Rate and Thermoregulation in Three Layered Human Skin during Carpentering, Swimming and Marathon, *Applied Mathematics*, *753-770* <https://www.scirp.org/journal/am>
- WHO (2020). Guidelines on physical activity and sedentary behavior
- Wiaro, G. (2013). Fisiologi dan olahraga. Yogyakarta: Graha Ilmu, 153-75.