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DEVELOPMENT OF A FORMULA FOR N-PERIOD DYNAMIC PROGRAMMING MODEL: AN APPLICATION OF ECONOMIC ORDER QUANTITY (EOQ)

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Abstract

This work analyzed an N-period dynamic programming planning horizon as applied to Economic Order Quantity (EOQ). The ordering policy during each period considers when demand is favorable or unfavorable to determine the EOQ and the associated profit at the end of each planning horizon. We also generate a formula to determine the number of matrix transitions for each planning horizon until final stage and then prove the formula by the principle of mathematical induction.

KEYWORDS:
Dynamic programming, EOQ and planning horizon.

INTRODUCTION

Inventory management is critical to the success of all supply chains. Many researchers have made great efforts to identify effective inventory policies to determine when and how much to order a product (Zipkin, 2000). Establishing an effective inventory policy often requires an in-depth analysis of the nature of the target business. Inventory management refers to decision making on the process of ordering, storing, and holding items in any company. This process includes the management of raw materials, components, and finished products, as well as warehousing and processing. Companies can use different inventory management methods according to the type of business or product involved. One of these management methods includes the EOQ.

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The EOQ model is used in inventory management by calculating the number of units a company should order or produce to replenish the inventory and minimizing the total inventory cost. EOQ can be a valuable tool for small business owners who need to make decisions about how much inventory to keep on hand, how many items to order each time, and how often to reorder. The model is widely used by practitioners as a decision-making tool for the control of inventory. In general, the objective of inventory management deals with maximization of profit or minimization of the inventory carrying cost. Therefore, it is important to determine the optimal stock and optimal time of replenishment of inventory to meet the future demand.

Dynamic programming (DP) is a mathematical modelling theory that is useful for solving a set of problems involving a sequence of interrelated decisions. DP provides a systematic means of solving multistage problems over a planning horizon or a sequence of probabilities. DP is a powerful tool that allows segmentation or decomposition of complex multistage problem into a number of simpler subproblems which are simpler to handle than the complete problem. Fundamental to this decomposition is the principle of optimality which was developed by Richard Bellman in the 1950s.

Abouee-Mehrizi et al. (2015) considered a finite horizon multi-period inventory system where the objective was to determine the optimal joint replenishment and transshipment policies, and found that the optimal ordering policy in each period was determined based on two switching curves.

Xu (2009) studied a single-product, dynamic, nonstationary, stochastic inventory problem with capacity commitment, in which a buyer purchases a fixed capacity from a supplier at the beginning of a planning horizon and the buyer's total cumulative order quantity over the planning horizon is constrained with the capacity. The objective of the buyer is to choose the capacity at the beginning of the planning horizon and the order quantity in each period to minimize the expected total cost over the planning horizon.

Lim (2011) proposed a stochastic nonlinear mixed binary integer programming model for a multi-period inventory problem with quantity discounts based on previous orders. The model examined quantity discount contracts between a manufacturer and a retailer in a stochastic, two-period inventory situation in which quantity discounts are provided on the basis of previous order size. Chen and Wei (2012) examined the problem of optimally dynamic joint decisions, including

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replenishment and quantity, retail and wholesale price and revenue-sharing allocation over multi-period planning horizon, subject to deteriorating goods and multivariant demand function. They used a calculus-based formulation combined with dynamic programming techniques to solve the channel coordination decision problem. Schmitt and Snyder (2010) developed an infinite-horizon inventory control model under both yield uncertainty and disruptions. They demonstrated the importance of analyzing a sufficiently long-time horizon when modeling inventory systems subject to supply disruptions and pointed out that using a single-period approximation could lead to a wrong strategy for mitigating supply risks.

Mubiru (2015) considered the inventory of a stochastic system whose aim was to optimize the order quantity and profits associated with ordering and holding cost of a single item. The decision whether to order additional units of the item or not to order was modelled based on two demand states, that is the favorable state and unfavorable state using dynamic programming.

Mubiru *et al.* (2017) considered a joint location inventory replenishment problem involving a chain of supermarkets at designated locations under stochastic demand. They formulated a finite state Markov decision process model where states of a Markov chain represent possible states of demand for milk powder product. The unit replenishment cost, shortage cost, demand and inventory positions were used to generate the total inventory cost matrix representing the long run measure of performance for the Markov decision problem. The replenishment decisions were made using dynamic programming over a finite period planning horizon. Mubiru and Ssempijja (2018) proposed a mathematical model that optimizes campaign strategies of a political candidate. Considering uncertainty in voter support and cost implications in holding political rallies, they formulated a finite state Markov decision process model where states of the Markov chain were represented by possible states of support among voters. The decision of whether or not to campaign and hold a political rally at a given location were made using discrete time Markov chains and dynamic programming over a finite period planning horizon. Results from the study indicated the existence of an optimal state-dependent campaign strategy and costs at the respective political rally locations.

Mubiru *et al.* (2019) considered a multi-period, single-item production-inventory lot sizing problem where demand is stochastic and non-stationary. They

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formulated a finite state Markov decision model where states of the Markov chain represent possible states of demand for the product. The decision of whether or not to produce additional inventory units is made using dynamic programming over a finite period planning horizon. The approach demonstrated the existence of an optimal state dependent production lot size (PLS) and produced an optimal lot sizing policy as well as the corresponding total production and inventory costs. Aisha and Sani (2020) modified the work of Mubiru (2015) by changing some parameters in the computation of demand transition and the profit matrix so as to reflect what is actually happening in real life. The model demonstrated an approach to optimize the EOQ of a single item under a periodic review inventory system with stochastic demand. They determined an optimal EOQ as well as the corresponding profits for each state of demand. A 2-period dynamic programming was used to determine the optimal inventory replenishment policy.

Aisha and Sani (2021) formulated a single item replenishment model with a periodic review inventory system under stochastic demand. The states of demand for the model were represented by Markov decision process with the objective to determine an optimal EOQ and corresponding profit in each period of the planning horizon. The replenishment policy was made using a 3- period dynamic programming technique.

MODEL PARAMETERS

C_h = Holding cost per unit

Q^z = Demand transition matrix for z policy

n, N = Stages going from n=1 to N

P_{i_n, j_n} = Profit in state

C_o = Ordering cost per unit

C_s = Shortage cost per unit

i_n , j_n = States of demand for n stage

C_p = Cost price per unit

$O_{i_n}^z$ = Economic order quantity

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D^z = Demand matrix for z policy

P^z = Profit matrix for z policy

P_s = Selling price per unit

z = Ordering policy

f = Favorable state

u = Unfavorable state

$D^z_{i_n, j_n}$ = Quantity demanded in state i_n, j_n for z policy

I^z = Inventory matrix for z policy

$e^z_{i_n}$ = Expected total profits at the end of the first period

$Q^z_{i_n, j_n}$ = Demand transition probability in state i_n, j_n for z policy

$I^z_{i_n, j_n}$ = Quantity in inventory in state i_n, j_n for z policy

a^z_{n,i_n} = Accumulated total profits at the end of the second period

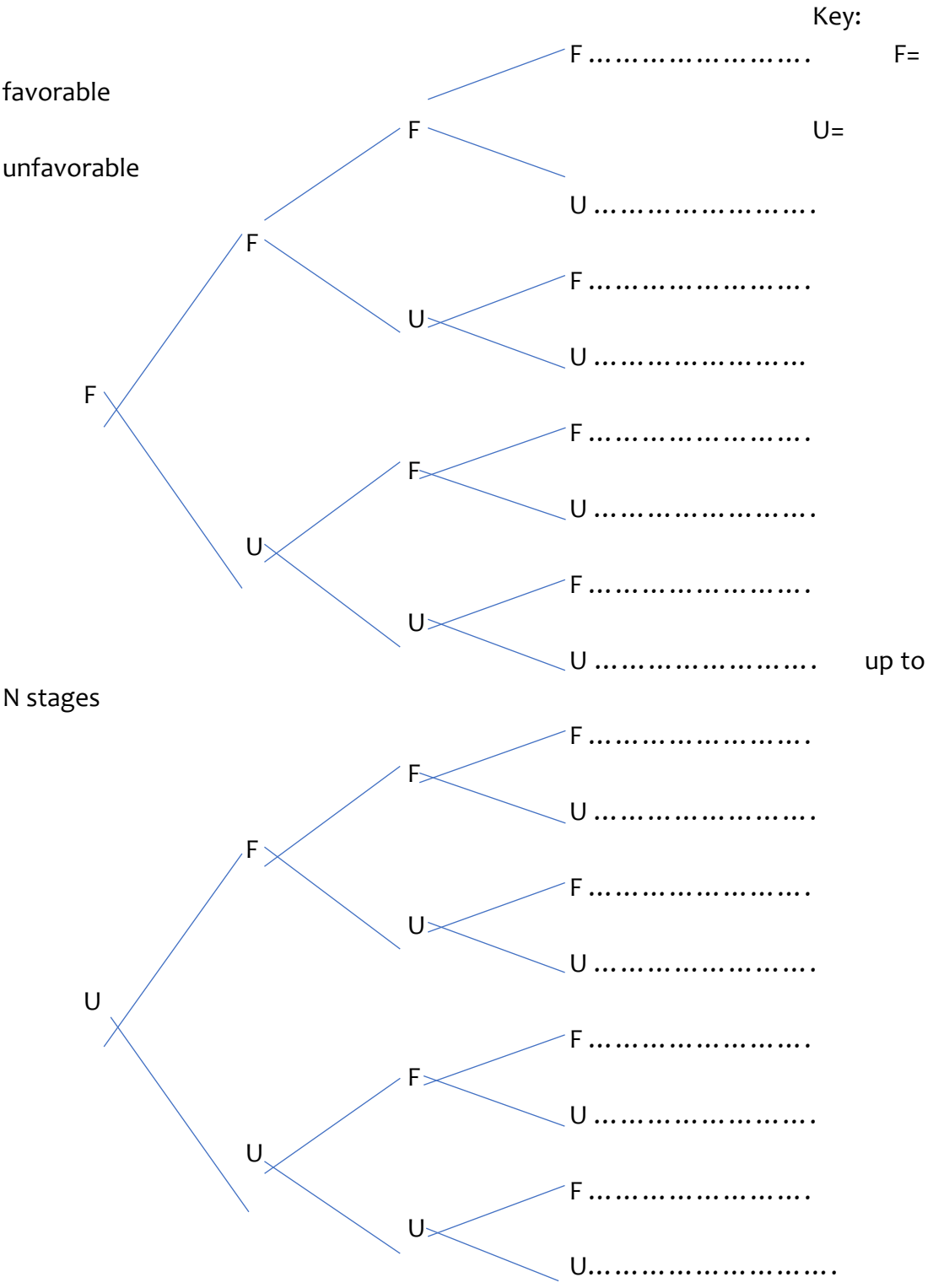
$g_n(i_n, p)$ = Optimal expected total inventory cost at stage n.

MODEL FORMULATION

This model considers an inventory system of a single item over a finite planning horizon. The demand for the item can be classified into favorable state and unfavorable state. The transition probabilities over the planning horizon can be expressed as Markov decision process. The decision to order additional units (z=1) or not to order additional units (z=0) is dependent on the state of demand where z is the decision variable. This decision is to be made using dynamic programming technique over N-period planning horizon. The decision tree diagram for the model is as follows:

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Key:
F= favorable
U= unfavorable
F
U
F
U
F
U

$$Q^{z}_{i_n,j_n} \Leftrightarrow \begin{bmatrix} Q^{z}_{fff\ldots f} & Q^{z}_{fff\ldots u} & Q^{z}_{ffu\ldots f} & \ldots\ldots & \ldots\ldots & \ldots\ldots & Q^{z}_{fuu\ldots f} & Q^{z}_{fuu\ldots u} \\ Q^{z}_{uff\ldots f} & Q^{z}_{uff\ldots u} & Q^{z}_{ufu\ldots f} & \ldots\ldots & \ldots\ldots & \ldots\ldots & Q^{z}_{uuu\ldots f} & Q^{z}_{uuu\ldots u} \end{bmatrix}$$

$$D^{z}_{i_n,j_n} \Leftrightarrow \begin{bmatrix} D^{z}_{fff\ldots f} & D^{z}_{fff\ldots u} & D^{z}_{ffu\ldots f} & \ldots\ldots & \ldots\ldots & \ldots\ldots & D^{z}_{fuu\ldots f} & D^{z}_{fuu\ldots u} \\ D^{z}_{uff\ldots f} & D^{z}_{uff\ldots u} & D^{z}_{ufu\ldots f} & \ldots\ldots & \ldots\ldots & \ldots\ldots & D^{z}_{uuu\ldots f} & D^{z}_{uuu\ldots u} \end{bmatrix}$$

$$I^{z}_{i_n,j_n} \Leftrightarrow \begin{bmatrix} I^{z}_{fff\ldots f} & I^{z}_{fff\ldots u} & I^{z}_{ffu\ldots f} & \ldots\ldots & \ldots\ldots & \ldots\ldots & I^{z}_{fuu\ldots f} & I^{z}_{fuu\ldots u} \\ I^{z}_{uff\ldots f} & I^{z}_{uff\ldots u} & I^{z}_{ufu\ldots f} & \ldots\ldots & \ldots\ldots & \ldots\ldots & I^{z}_{uuu\ldots f} & I^{z}_{uuu\ldots u} \end{bmatrix}$$

$$P^{z}_{i_n,j_n} \Leftrightarrow \begin{bmatrix} P^{z}_{fff\ldots f} & P^{z}_{fff\ldots u} & P^{z}_{ffu\ldots f} & \ldots\ldots & \ldots\ldots & \ldots\ldots & P^{z}_{fuu\ldots f} & P^{z}_{fuu\ldots u} \\ P^{z}_{uff\ldots f} & P^{z}_{uff\ldots u} & P^{z}_{ufu\ldots f} & \ldots\ldots & \ldots\ldots & \ldots\ldots & P^{z}_{uuu\ldots f} & P^{z}_{uuu\ldots u} \end{bmatrix}$$

Converting to multiple transition matrices, we obtain

$$Q^{z}_{i_n,j_n} = \begin{bmatrix} Q^{z}_{ff} & Q^{z}_{fu} \\ Q^{z}_{uf} & Q^{z}_{uu} \end{bmatrix}\begin{bmatrix} Q^{z}_{ff} & Q^{z}_{fu} \\ Q^{z}_{uf} & Q^{z}_{uu} \end{bmatrix}\begin{bmatrix} Q^{z}_{ff} & Q^{z}_{fu} \\ Q^{z}_{uf} & Q^{z}_{uu} \end{bmatrix}\ldots\ldots\begin{bmatrix} Q^{z}_{ff} & Q^{z}_{fu} \\ Q^{z}_{uf} & Q^{z}_{uu} \end{bmatrix} = \begin{bmatrix} Q^{z}_{FF} & Q^{z}_{FU} \\ Q^{z}_{UF} & Q^{z}_{UU} \end{bmatrix} \text{ say}$$

$$D^{z}_{i_n,j_n} = \begin{bmatrix} D^{z}_{ff} & D^{z}_{fu} \\ D^{z}_{uf} & D^{z}_{uu} \end{bmatrix}\begin{bmatrix} D^{z}_{ff} & D^{z}_{fu} \\ D^{z}_{uf} & D^{z}_{uu} \end{bmatrix}\begin{bmatrix} D^{z}_{ff} & D^{z}_{fu} \\ D^{z}_{uf} & D^{z}_{uu} \end{bmatrix}\ldots\ldots\begin{bmatrix} D^{z}_{ff} & D^{z}_{fu} \\ D^{z}_{uf} & D^{z}_{uu} \end{bmatrix} = \begin{bmatrix} D^{z}_{FF} & D^{z}_{FU} \\ D^{z}_{UF} & D^{z}_{UU} \end{bmatrix} \text{ say}$$

$$I^{z}_{i_n,j_n} = \begin{bmatrix} I^{z}_{ff} & I^{z}_{fu} \\ I^{z}_{uf} & I^{z}_{uu} \end{bmatrix}\begin{bmatrix} I^{z}_{ff} & I^{z}_{fu} \\ I^{z}_{uf} & I^{z}_{uu} \end{bmatrix}\begin{bmatrix} I^{z}_{ff} & I^{z}_{fu} \\ I^{z}_{uf} & I^{z}_{uu} \end{bmatrix}\ldots\ldots\begin{bmatrix} I^{z}_{ff} & I^{z}_{fu} \\ I^{z}_{uf} & I^{z}_{uu} \end{bmatrix} = \begin{bmatrix} I^{z}_{FF} & I^{z}_{FU} \\ I^{z}_{UF} & I^{z}_{UU} \end{bmatrix} \text{ say}$$

$$P^{z}_{i_n,j_n} = \begin{bmatrix} P^{z}_{ff} & D^{z}_{fu} \\ P^{z}_{uf} & D^{z}_{uu} \end{bmatrix}\begin{bmatrix} P^{z}_{ff} & P^{z}_{fu} \\ P^{z}_{uf} & P^{z}_{uu} \end{bmatrix}\begin{bmatrix} P^{z}_{ff} & P^{z}_{fu} \\ P^{z}_{uf} & P^{z}_{uu} \end{bmatrix}\ldots\ldots\begin{bmatrix} P^{z}_{ff} & P^{z}_{fu} \\ P^{z}_{uf} & P^{z}_{uu} \end{bmatrix} = \begin{bmatrix} P^{z}_{FF} & P^{z}_{FU} \\ P^{z}_{UF} & P^{z}_{UU} \end{bmatrix} \text{ say}$$

In carrying out the work, we apply the modifications in Hassan and Sani (2021) done to the work of Mubiru (2015).

Note that, the number of matrices that make up the right-hand matrix are calculated from the formula below: that is, Number of transitions matrices at N stage where N is finite.

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$$\sum_{n=0}^{N-2} 2^n = 2^{N-1} - 1 \qquad (1.1)$$

Thus, since N is the number of periods which is the same as the planning horizon, the transition has to start from 2. If I_N is the number of transition matrices at N stage, then we have:

$$I_2 = \sum_{n=0}^{0} 2^n = 2^{2-1} - 1 = 2 - 1 = 1$$

$$(1.2)\ I_3 = \sum_{n=0}^{1} 2^n = 2^{3-1} - 1 = 2^2 - 1 = 3$$

(1.3)

$$I_4 = \sum_{n=0}^{2} 2^n = 2^{4-1} - 1 = 2^3 - 1 = 7$$

(1.4) and so on.

Proof of the formula

To prove the formula above, we note that we start with f and u. In the first transition, f may transit to another f or to u. On the other hand, u may transit to f or to u also. This gives a single transition matrix $\begin{bmatrix} ff & fu \\ uf & uu \end{bmatrix}$. Therefore $I_2 = 1.$

In the next transition, ff may transit to fff or to ffu. On the other hand, fu may also transit to fuf or to fuu which will give another transition matrix similar to the initial one. In the same way, uf may transit to uff or to uuu giving another transition matrix similar to the initial one as well. This makes three multiple transition matrices, and so on. Thus $I_3 = 3$ and we can use the principle of mathematical induction to prove the formula as follows:

For $N = 2,\ I_2 = 2^{2-1} - 1 = 1$. Therefore, the result is true for the first N which is equal 2.

Next, we assume that the result is true for $N = k$ i.e. $I_k = 2^{k-1} - 1$. That is, $I_K = 2^0 + 2^1 + 2^2 + \ldots\ldots\ldots\ldots + 2^{k-2} = 2^{k-1} - 1.$ Then for $N = k+1$ we have

$$I_{k+1} = 2^0 + 2^1 + 2^2 + \ldots\ldots\ldots\ldots + 2^{k-2} + 2^{k-1}$$

$$= \sum_{n=0}^{k-2} 2^n + 2^{k-1}$$

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$= 2^{k-1} - 1 + 2^{k-1} = 2(2^{k-1}) - 1$

$= 2^k - 1$

Hence, the result is true for $N = k+1$ and so the result is true for any N such that $N \geq 2$. Therefore, the result is proved.

DYNAMIC PROGRAMMING FORMULATION

We consider an inventory system of a single item with two states of demand. That is, the favorable state (f) and the unfavorable state (u) based on the situation of the market. The problem can be expressed as a finite-period dynamic programming model to obtain an optimal EOQ. Denoted by $g_n(i_n)$ the expected total profit accumulated during the n periods for $n \leq N$ given that the state of the system is $i_n \in \{f, u\}$ at the start of period n

The recursive equation relating g_n and g_{n+1} is as follows

$$g_n(i_n) = \max_z \{Q^z_{i_n f}(P^z_{i_n f} + g_{n+1}(f)), Q^z_{i_n u}(P^z_{i_n u} + g_{n+1}(u))\}$$

(1.5)

$$z \in \{0,1\},\ i_n \in \{f, u\}$$

$$n = 1, 2.....N$$

for $$g_N(f) = g_N(u) = 0$$

(1.6)

where the cumulative total profit is given by

$$P^z_{i_n, j_n} + g_{n+1}(j_n)$$

(1.7)

As a result of getting to state $j_n \in \{f, u\}$ at the start of period n+1 from state $i_n \in \{f, u\}$ at the initial period n, occurring with the probability $Q^z_{i_n, j_n}$.

This shows that $e^z_{i_n} = Q^z (P^z)^T$.

(1.8)

Then the dynamic recursive equation becomes

$$g_n(i_n) = \max_z \{e^z_{i_n} + Q^z_{i_n f} g_{n+1}(f) + Q^z_{i_n u} g_{n+1}(u)\}$$

(1.9)

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or $g_N(i_n) = \max_z \left[e_{i_n}^z \right]$

(2.0)

COMPUTING THE ECONOMIC ORDER QUANTITY FOR THE PLANNING HORIZONS

We consider an N-period planning horizon in this paper and so the planning starts from $n = 1, 2, 3.....N-1, N$, as follows:

Optimization during the first planning horizon

The ordering policy during period 1 when the demand is favorable

$$z = \begin{cases} 1, & if\ e_f^1 > e_f^0 \\ 0, & if\ e_f^1 \le e_f^0 \end{cases} \qquad (2.1)$$

The associated total profit and EOQ are

$$g_1(f) = \begin{cases} e_f^1, & if\ z = 1 \\ e_f^0, & if\ z = 0 \end{cases} \qquad (2.2)$$

and $$O_f^z = \begin{cases} (D_{ff}^1 - I_{ff}^1) + (D_{fu}^1 - I_{fu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases}$$

(2.3)

The ordering policy during period 1 when the demand is unfavorable

$$z = \begin{cases} 1, & if\ e_u^1 > e_u^0 \\ 0, & if\ e_u^1 \le e_u^0 \end{cases}$$

(2.4)

The associated total profit and EOQ are

$$g_1(u) = \begin{cases} e_u^1, & if\ z = 1 \\ e_u^0, & if\ z = 0 \end{cases}$$

(2.5)

and

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$$O_u^z = \begin{cases} (D_{uf}^1 - I_{uf}^1) + (D_{uu}^1 - I_{uu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases}$$

(2.6)

Optimization during the second planning horizon

a_n^z represent the accumulated profits at the end of the first period as a result of decisions made using the equation below:

$$a_1^z = e_i^z + Q_{if}^z g_1(f) + Q_{iu}^z g_1(u) \qquad (2.7)$$

Therefore, the ordering policy when the demand is favorable is

$$z = \begin{cases} 1, & if\ a_{1f}^1 > a_{1f}^0 \\ 0, & if\ a_{1f}^1 \le a_{1f}^0 \end{cases}$$

(2.8)

The associated total profits and EOQ are

$$g_2(f) = \begin{cases} a_{1f}^1, & if\ z = 1 \\ a_{1f}^0, & if\ z = 0 \end{cases}$$

(2.9)

and

$$O_f^z = \begin{cases} (D_{ff}^1 - I_{ff}^1) + (D_{fu}^1 - I_{fu}^1); & if\ z = 1 \\ 0; & if\ z = 0 \end{cases}$$

(3.0)

Also, the ordering policy when the demand is unfavorable is

$$z = \begin{cases} 1, & if\ a_{1u}^1 > a_{1u}^0 \\ 0, & if\ a_{1u}^1 \le a_{1u}^0 \end{cases}$$

(3.1)

The associated total profits and EOQ are

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$$g_2(u) = \begin{cases} a_{1u}^1, \ if \ z = 1 \\ a_{1u}^0, \ if \ z = 0 \end{cases}$$

(3.2)

and

$$O_u^z = \begin{cases} (D_{uf}^1 - I_{uf}^1) + (D_{uu}^1 - I_{uu}^1); \ \ if \ z = 1 \\ 0; \ if \ z = 0 \end{cases}$$

(3.3)

Optimization during the third-period planning horizon

The accumulated profits at the end of period 3 as a result of decisions made using the equation as follows:

$$a_2^z = a_1^z + Q_{if}^z g_2(f) + Q_{iu}^z g_2(u) \qquad (3.4)$$

Therefore, the ordering policy when the demand is favorable is

$$z = \begin{cases} 1, \ \ if \ a_{2f}^1 > a_{2f}^0 \\ 0, \ \ if \ \ a_{2f}^1 \le a_{2f}^0 \end{cases}$$

(3.5)

The associated total profits and EOQ are

$$g_3(f) = \begin{cases} a_{2f}^1, \ if \ z = 1 \\ a_{2f}^0, \ if \ z = 0 \end{cases}$$

(3.6)

and

$$O_f^z = \begin{cases} (D_{ff}^1 - I_{ff}^1) + (D_{fu}^1 - I_{fu}^1); \ \ if \ z = 1 \\ 0; \ if \ z = 0 \end{cases}$$

(3.7)

Also, the ordering policy when the demand is unfavorable is

$$z = \begin{cases} 1, \ \ if \ a_{2u}^1 > a_{2u}^0 \\ 0, \ if \ a_{2u}^1 \le a_{2u}^0 \end{cases}$$

(3.8)

The associated total profits and EOQ are

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$$g_3(u)=\begin{cases} a_{2u}^{1},\ if\ z=1 \\ a_{2u}^{0},\ if\ z=0 \end{cases}$$

(3.9)

and

$$O_u^z=\begin{cases} (D_{uf}^1-I_{uf}^1)+(D_{uu}^1-I_{uu}^1);\ \ if\ z=1 \\ 0;\ if\ z=0 \end{cases}$$

(4.0)

Computing this way up to N-1 period, we get

Optimization during (N-1)-period planning horizon

The accumulated profits at the end of (N-1)-period as a result of decisions made using the equation as follows:

$$a_{N-2}^{z}=a_{N-3}^{z}+Q_{if}^{z}g_{N-2}(f)+Q_{iu}^{z}g_{N-2}(u)$$

(4.1)

Therefore, the ordering policy when the demand is favorable is

$$z=\begin{cases} 1,\ \ if\ a_{N-2f}^{\ 1}>a_{N-2f}^{\ 0} \\ 0,\ \ if\ \ a_{N-2f}^{\ 1}\le a_{N-2f}^{\ 0} \end{cases}$$

(4.2)

The associated total profits and EOQ are

$$g_{N-1}(f)=\begin{cases} a_{N-2f}^{\ 1},\ if\ z=1 \\ a_{N-2f}^{\ 0},\ if\ z=0 \end{cases}$$

(4.3)

and

$$O_f^z=\begin{cases} (D_{ff}^1-I_{ff}^1)+(D_{fu}^1-I_{fu}^1);\ \ if\ z=1 \\ 0;\ if\ z=0 \end{cases}$$

(4.4)

Also, the ordering policy when demand is unfavorable is

$$z=\begin{cases} 1,\ \ if\ a_{N-2u}^{\ 1}>a_{N-2u}^{\ 0} \\ 0,\ if\ a_{N-2u}^{\ 1}\le a_{N-2u}^{\ 0} \end{cases}$$

(4.5)

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The associated total profits and EOQ are

$$g_{N-1}(u)=\begin{cases} a_{N-2u}^{\ 1}, \ if \ z=1 \\ a_{N-2u}^{\ 0}, \ if \ z=0 \end{cases}$$

(4.6)

and

$$O_u^z=\begin{cases} (D_{uf}^1-I_{uf}^1)+(D_{uu}^1-I_{uu}^1); \ \ if \ z=1 \\ 0; \ if \ z=0 \end{cases}$$

(4.7)

Optimization during N-period planning horizon

The accumulated profits at the end of N-period as a result of decisions made using the equation as follows:

$$a_{N-1}^z=a_{N-2}^z+Q_{if}^z g_{N-1}(f)+Q_{iu}^z g_{N-1}(u)$$

(4.8)

Therefore, the ordering policy when the demand is favorable is

$$z=\begin{cases} 1, \ \ if \ a_{N-2f}^{\ 1} > a_{N-2f}^{\ 0} \\ 0, \ \ if \ \ a_{N-2f}^{\ 1} \le a_{N-2f}^{\ 0} \end{cases}$$

(4.9)

The associated total profits and EOQ are

$$g_{N-1}(f)=\begin{cases} a_{N-2f}^{\ 1}, \ if \ z=1 \\ a_{N-2f}^{\ 0}, \ if \ z=0 \end{cases}$$

(5.0)

and

$$O_f^z=\begin{cases} (D_{ff}^1-I_{ff}^1)+(D_{fu}^1-I_{fu}^1); \ \ if \ z=1 \\ 0; \ if \ z=0 \end{cases}$$

(5.1)

Also, the ordering policy when the demand is unfavorable is

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$$z = \begin{cases} 1, \ if \ a_{N-2u}^{\ 1} > a_{N-2u}^{\ 0} \\ 0, \ if \ a_{N-2u}^{\ 1} \le a_{N-2u}^{\ 0} \end{cases}$$

(5.2)

The associated total profits and EOQ are

$$g_{N-1}(u) = \begin{cases} a_{N-2u}^{\ 1}, \ if \ z = 1 \\ a_{N-2u}^{\ 0}, \ if \ z = 0 \end{cases}$$

(5.3)

and

$$O_u^z = \begin{cases} (D_{uf}^1 - I_{uf}^1) + (D_{uu}^1 - I_{uu}^1); \ \ if \ z = 1 \\ 0; \ if \ z = 0 \end{cases}$$

(5.4)

CONCLUSION

In this work, we develop a model for N-period dynamic programming technique under a periodic review inventory system with stochastic demand. With the aid of dynamic programming, we analyze the N-period planning horizons for a single item inventory system with favorable and unfavorable states of demand where N is finite. We also generate the formula for obtaining the number of matrix transitions from one state to another and we proved the formula by the principles of mathematical induction.

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